

## Velocity model determination in seismic lines via traveltime inversion in the Ahwaz oil field

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### Abstract

Traveltime inversion has been used in this study in order to obtain a velocity model. This velocity model has been used in a depth migration algorithm afterwards.

To achieve this purpose three main steps are as follows: picking phases, forward modeling, inverse modeling and finally using an appropriate depth migration algorithm to apply to the data.

**Key words:** Depth migration, Traveltime inversion, Modeling, Picking phase, Velocity model

### 1 INTRODUCTION

Seismic imaging is changing the face of the geophysics industry by combining the two separate parts of processing and interpretation. It is used to image not only complex structures but also subtle geological structures. It is helpful in two ways. Firstly by velocity positioning and secondly by lateral positioning.

### 2 PHASE PICKING

A seismic phase can be defined as a set of arrivals in a seismic section which has its origin in one layer of assumed homogeneous physical properties. The arrivals should all have the same kind of origin, i. e. a reflection, a refraction or a headwave.

A quality measure for a seismic phase is its degree of correlation or coherency. Coherency is a measure of the similarity of a trace with the neighboring traces, i.e. how similar the amplitudes are at similar traveltimes in the neighboring traces. Programs that search a section for correlated energy will find many events, and it is up to the interpreter to decide which of them should be used in further processing.

Automatic line drawing programs scan a section for correlated events. The following parameters can usually be varied: the maximum dip of the correlated event, its spatial extension, the correlation levels to be taken into account and the size of the data window the search is performed in. Thus, the automatic line drawing can be of additional help for the phase identification. It gives objective values of both coherency and amplitude of the identified events. On the other hand, no information on the character and significance of the events is given.

Especially multiples or reverberations give rise to correlated events that cannot contribute to the modeling. This has to be taken into account when the significant phases are selected. The modeling concept plays an important role since the way the model is parameterized determines what phases can be used.

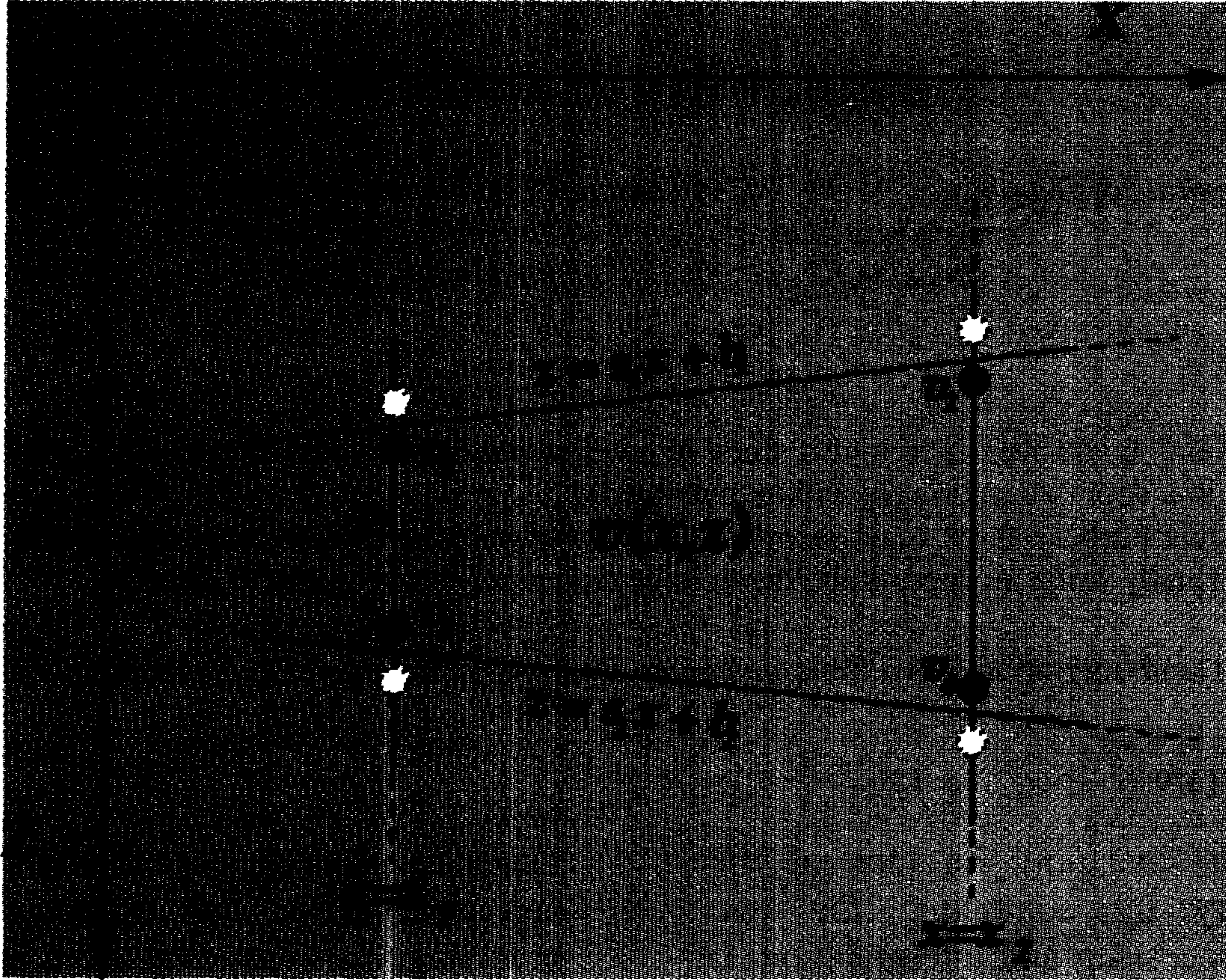
The phases picked from the seismic sections are the input data used in the modeling. Sensible picking is therefore vital for the further progress and the outcome of the work. Picking can be done automatically or by hand, and the choice of method to use depends on the modeling procedure applied later. In this work the phases were picked manually on the computer screen. A total number of 5280 traveltimes were picked.

### 3 FORWARD MODELING

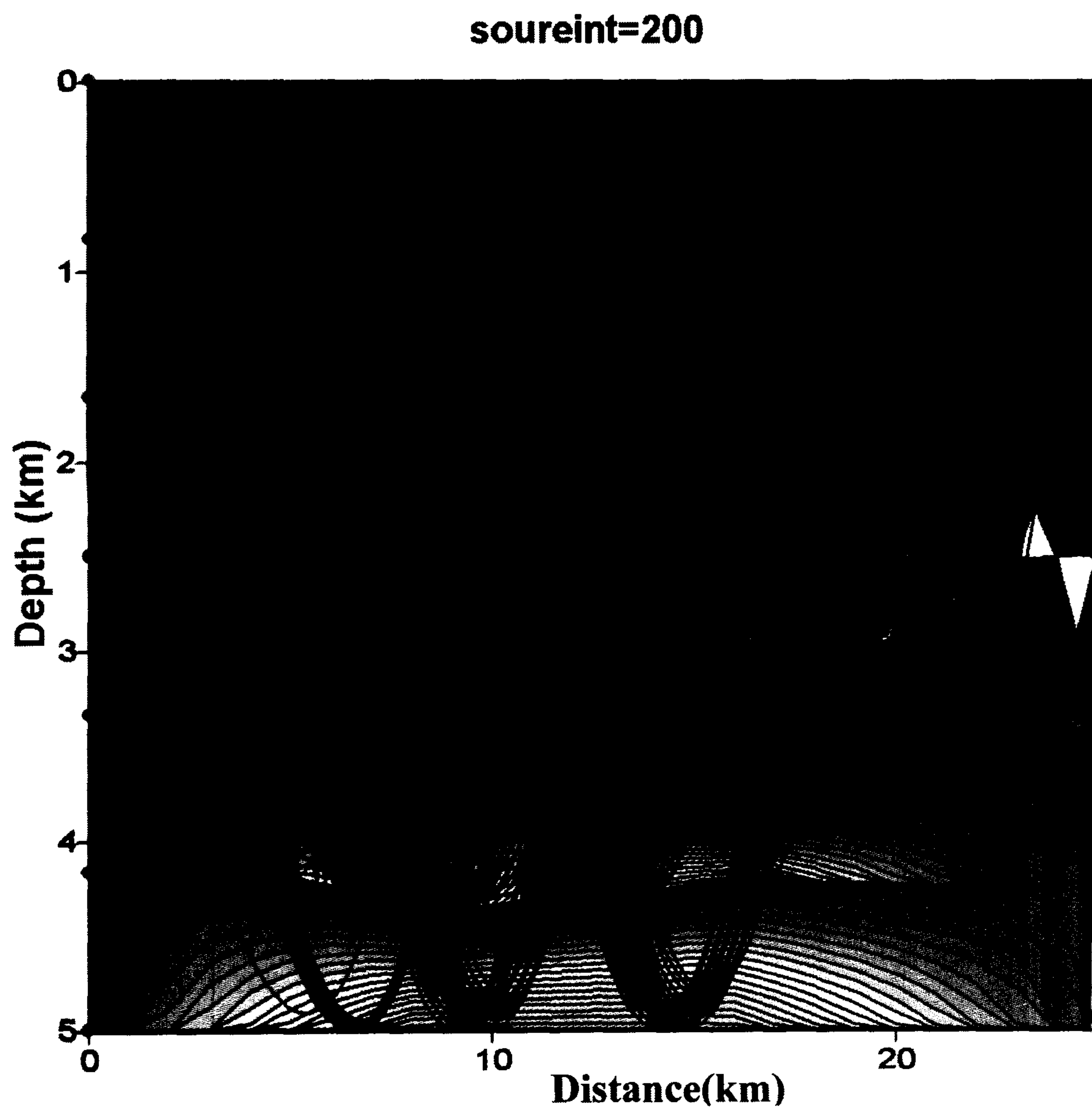
Since no fully developed automatic modeling approach is available, a more conventional method was used in this work. It uses the concept of layered media. The modeling software used here, is xrayinvr, that consists of a forward modeling and inversion algorithm which was developed and described by Zelt and Smith (1992). It is based on an old program, ray84, but with more interactive and X-windows graphics functions added.

The theoretical background of the program which it is based on is explained by Cerveny et al. (1977). The model is divided into a number of layers in which the velocities are defined at the corners of trapezoidal cells. The velocities at every point within each cell are interpolated after the following formula (see also Figure 1 for a graphical explanation):

$$x = x_1, x = x_2, z = s_1x + b_1, z = s_2x + b_2 \quad (1)$$



**Figure 1.** Geometry of the velocity node coordinates. The velocities are defined at the corner points, and the depth values are calculated according to the starting points and the gradient given (Zelt, 1993).



**Figure 2.** Ray tracing in a 6-layered model. (\* Colour print on P. 54)

$$\begin{aligned}
c_1 &= s_2(x_2v_1 - x_1v_2) + b_2(v_2 - v_1) - \\
&\quad s_1(x_2v_3 - x_1v_4) - b_1(v_4 - v_3) \\
c_2 &= s_2(v_2 - v_1) - s_1(v_4 - v_3) \\
c_3 &= x_1v_2 - x_2v_1 + x_2v_3 - x_1v_4 \\
c_4 &= v_1 - v_2 + v_4 - v_3 \\
c_5 &= b_2(x_2v_1 - x_1v_2) - b_1(x_2v_3 - x_1v_4) \\
c_6 &= (s_2 - s_1)(x_2 - x_1) \\
c_7 &= (b_2 - b_1)(x_2 - x_1)
\end{aligned} \tag{2}$$

$$V_{(x,z)} = \frac{c_1x + c_2x^2 + c_3z + c_4xz + c_5}{c_6x + c_7} \tag{3}$$

In order to avoid the scattering of rays at points where a layer boundary changes its dip, a smoothing operator can simulate "smooth" interfaces. Since normally the number of nodes is lower than the resolution of the model and therefore the layer boundaries are rougher than desired, most of the scattering effects are not desired.

Other parameters needed by the program are the picks and a parameter file that contains all information on where in the model the rays are reflected, refracted or along which layer boundary the head waves propagate, how the model is plotted etc, Zelt (1993). The traveltimes picks can be coded according to the layer they turn in (refracted rays) or the boundary they are reflected at (reflected waves) or traveling along (head waves), and according to the pick uncertainty. One additional file contains information on the general velocity uncertainty; the boundary depth uncertainty and the damping factor used in the inversion (see next subsection).

When the picked phases are coded and an initial model is specified, forward modeling can begin. The program tries to trace all rays according to the specifications in the control file, and the result is plotted. If no ray of a phase can be traced, the program reports a warning.

Additionally, the rms traveltimes residual and the  $\chi^2$  value as objective measures of the model fit, are output.

The values for  $\Delta_{rms}$  and  $\chi^2$  are calculated as follows:

$$\Delta_{rms} = \sqrt{\frac{\sum_{i=1}^n (d_{i0} - d_{ic})^2}{n}} \tag{4}$$

( $n$  number of data points,  $d_{i0}$   $i$  th observed value,  $d_{ic}$   $i$  th calculated parameter)

$$\chi^2 = \frac{1}{n} \frac{\sum_{i=1}^n (d_{i0} - d_{ic})^2}{\sigma_i^2} \tag{5}$$

( $\sigma_i^2$  variance of the  $i$  th value)

Whereas the  $\Delta_{rms}$  value gives the general quality of the fit, the  $\sigma_i^2$  gives more information on the error distribution. If the model fits the observed data well  $\Delta_{rms}$ , lies close to zero. Since the value of  $\chi^2$  depends on the standard deviation, it gives more information on the error distribution. If  $\chi^2$  is close to unity, the error distribution is close to normal (provided that the variances are estimated correctly), but if the value deviates considerably from unity, the model contains systematic errors.

The model fit is improved by editing appropriate values in the parameter file, i.e. both velocities and layer depths that affect the traveltimes of the rays involved. If rays from different shots cross or overlay each other, the velocities and depths are better determined than in the case of parallel rays from one shot. Rays from independent sources are linearly independent.

In order to build an initial model, seismic data was stacked and the velocity model was created based on the stacked section (Figure 2).

#### 4 INVERSION AND DEPTH MIGRATION

Forward modeling is a very useful tool for creating a realistic starting model with the appropriate layers and realistic velocities. Inversion is used to optimize the fit of the model to the observed data automatically. This subsection gives a short introduction to the theory on which the algorithm is based.

As shown in Equation 6 the traveltimes is described by an integral along the ray path. Since the ray path along which the integration is performed depends on the velocity structure itself, the problem is non-linear. An iterative scheme based on linearization is required to solve the problem.

With  $n$  traveltimes observations we obtain a traveltimes vector  $t$ . First an initial traveltimes  $t_0^i$  is calculated:

$$t_i^0 = \int_{L_i} \frac{1}{v(x,z)} dl \tag{6}$$

along the  $i$  th ray path  $L_i$  in the initial model  $m^0$ . The  $i$  th traveltimes as a function of adjusted model parameters can be approximated by a Taylor series that is truncated after the first order term:

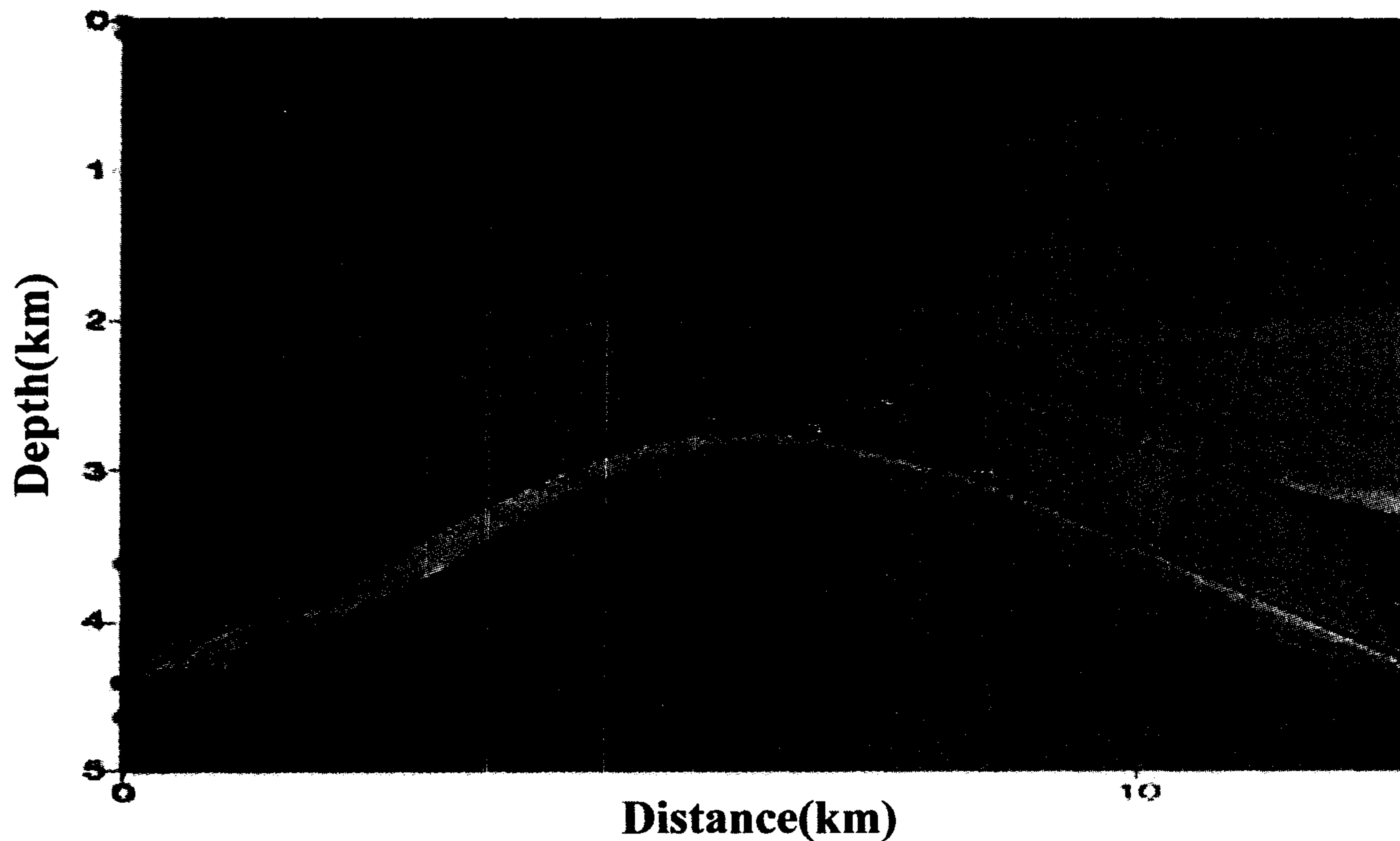


Figure 3. Initial velocity model of line 1131. (\* Colour print on P. 54)

$$t_i \cong t_i^0 + \sum_{j=1}^k \frac{\partial t_i}{\partial m_j} \Delta m_j \quad (7)$$

(  $m_{i,j} = 1, 2, \dots, k$  model parameters, i.e. reflector depths or velocities,  $m_j^0$  model parameter of the initial model,  $\Delta m_j = m_j - m_j^0$  adjustment of the initial model). (Figure 3) Hence, the traveltime perturbation  $\Delta t_i$  is the difference between observed and calculated traveltime:

$$\Delta t_i = t_i - t_i^0 \quad (8)$$

For all observed traveltimes, this leads to a system of linear equations, which can be expressed in matrix form:

$$\Delta t = A \Delta m \quad (9)$$

(  $\Delta t = (\Delta t_1, \Delta t_2, \dots, \Delta t_n)^T$  vector of traveltime residuals,  $A = A(ij)$  partial derivative matrix,

$$A_{ij} = \frac{\partial t_i}{\partial m_j}, \quad \Delta m = (\Delta m_1, \dots, \Delta m_k)^T \text{ model}$$

parameter adjustment vector)

The matrix  $A$  has  $n$  columns (the number of observed traveltimes) and  $k$  rows (the number of model parameters). The vector  $\Delta m$ , is calculated after the ray tracing and applied to the actual model. After that, the ray tracing is done in the updated model. This iterative adjustment process is repeated until the model fits the observed data satisfyingly (See Table 1) with the numerical output from the iterations).

The inverse problem formulated in Equation 9 is usually mixed-determined in wide-angle seismics, which means that some of the model parameters are over determined and others underdetermined. Including estimates for the model parameter covariance in the model covariance matrix  $C_m$  and the picking uncertainty of the data, described by the data covariance matrix  $C_t$  and an overall damping factor  $D$  yields maximum likelihood estimate for the adjustment vector (Menke, 1989):

$$\Delta m = (A^T C_t^{-1} A + D C_m^{-1})^{-1} A^T C_t^{-1} \Delta t \quad (10)$$

The traveltime uncertainties are specified for every pick individually, and the velocity and boundary depth uncertainties are general parameters valid for the whole model. The damping factor  $D$  is a scalar, chosen empirically, so that a good balance is found between data fit and model smoothness. The value of  $D$  can play an important role for the speed with which error minima are found in the parameter space. The determination of the traveltime error is done by estimation of data and pick quality in connection with the shot to receiver distance. General a priori errors in model parameter determination are empiric values.

The model resolution matrix can also be calculated with the provided parameters:

$$R = (A^T C_t^{-1} A + D C_m^{-1})^{-1} A^T C_t^{-1} A \quad (11)$$

Where  $C_t$  and  $C_m$  are the estimated data and model covariance matrices. The model resolution matrix elements range from 0 to 1, and its

diagonal elements indicate by how many rays a model cell is crossed. It is a relative measure dependent on the choice of the model covariance matrix. Tests with synthetic and real data have shown that a diagonal element value above 0.5 implies that the corresponding model parameter can generally be regarded as well resolved (Zelt, 1993).

A few practical issues must be taken into account in order to stabilize the iterative inversion process. The algorithm is not very sophisticated in a geophysical sense. The inversion can produce model parameters such as strong positive or negative velocity gradients or steep layer boundary dips that are unrealistic from a geophysical point of view. The solution space may contain many local minima in  $\Delta_{rms}$ , which can prevent the inversion from finding better

solutions. Before the inversion is run, a model is needed that lies close to the optimum model that can be achieved by forward modeling. In other words, considerable forward modeling work must be done before the inversion can work properly.

The inversion needs the vector of traveltime residuals  $\Delta t$  from one forward modeling run of the xrayinvr program. The vector is saved in a file and read by the damped least square inversion program dmplstsqr. After one inversion run, the velocity model is updated, and update information is output (Figure 4). This information needs to be controlled carefully for improper changes (see the previous paragraph). If unwanted changes are produced by the inversion, several parameters can be modified to make the process run differently. The initial model parameter can be changed in order to make the inversion find a different adjustment.

**Table 1.** Inversion parameters of line 1131.

Line 1131			
Number of Iterations	Damping Factor	$\chi^2$	$\Delta_{rms}$
0	15	21.526	0.232
1	15	13.152	0.181
2	15	10.028	0.158
3	15	8.825	0.158
5	15	8.352	0.155
5	15	7.830	0.150
6	10	7.577	0.138
7	10	7.222	0.135
8	10	6.598	0.132
9	10	6.775	0.130
10	5	6.315	0.126
11	5	5.926	0.122
12	5	5.660	0.119
13	5	5.580	0.117
15	1	5.009	0.112
15	1	5.627	0.108
16	5	6.822	0.131
17	5	6.570	0.127
18	5	6.305	0.126
19	5	6.226	0.125
20	1	6.083	0.123
21	1	5.812	0.121
22	1	5.671	0.119
23	1	5.391	0.116
25	1	5.801	0.110
25	1	5.555	0.107
26	1	5.282	0.103

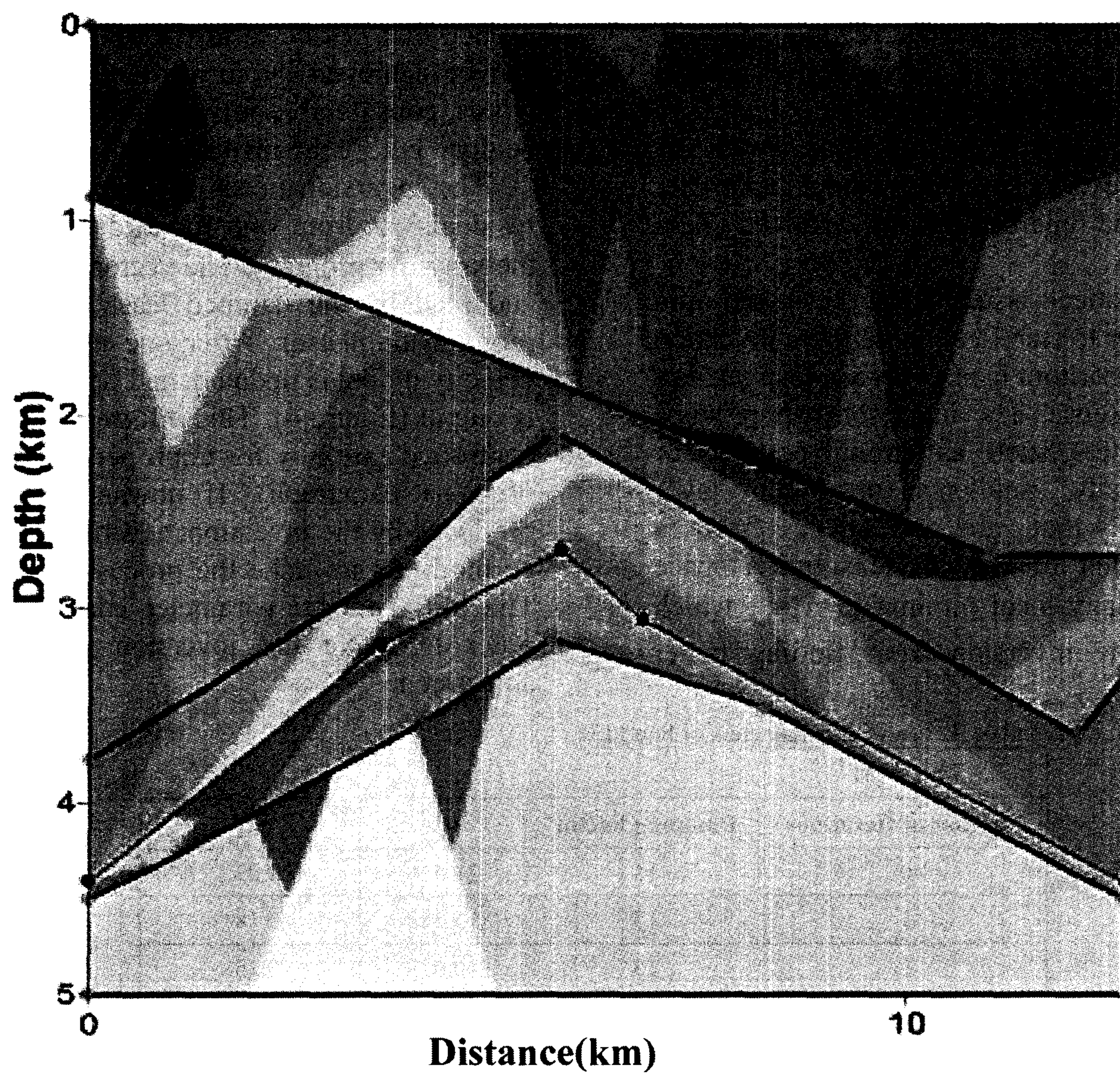


Figure 4. Inversion result of line 1131. (\* Colour print on P. 54)

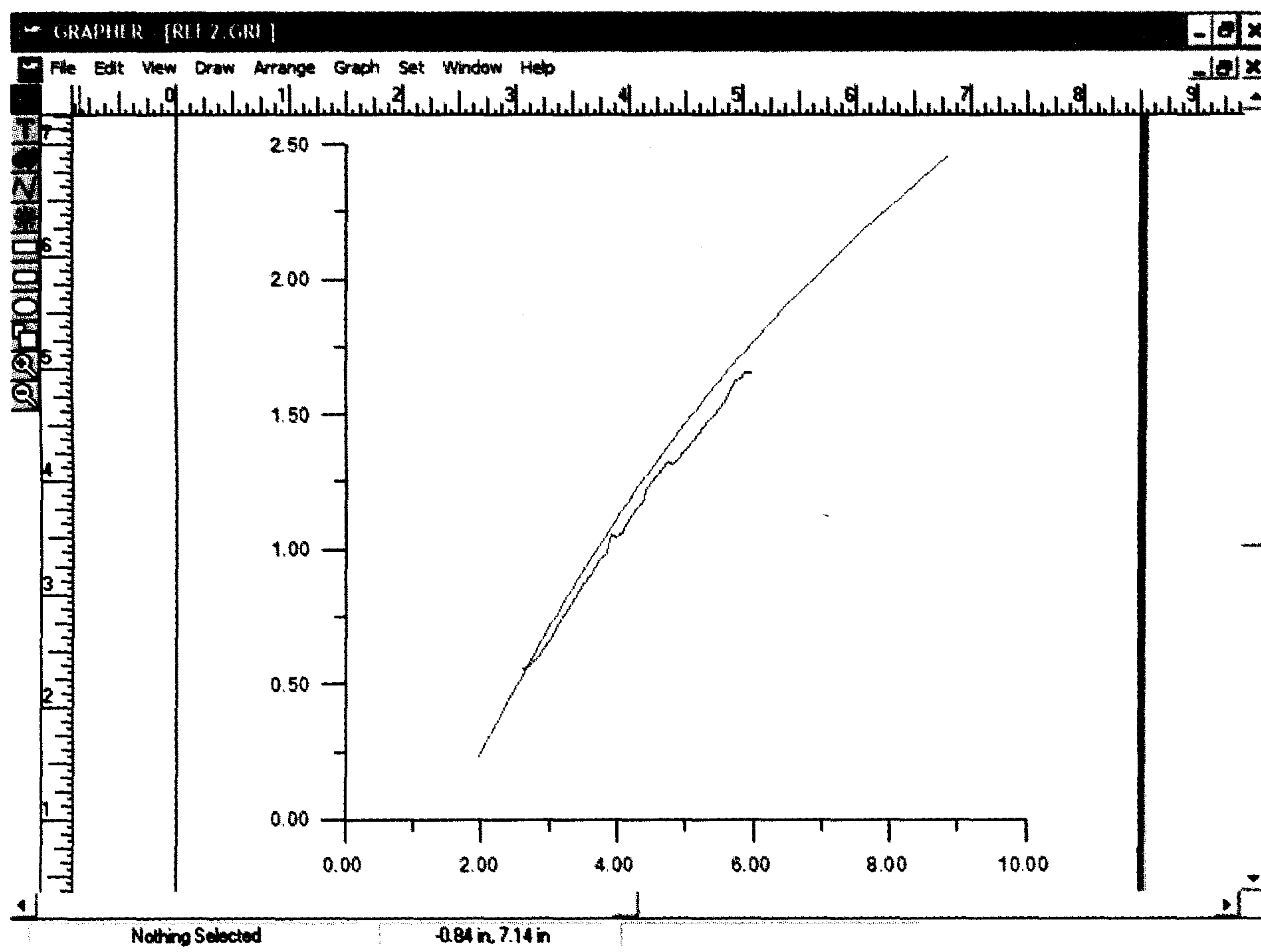


Figure 5. Data misfit after travelt ime inversion. Red curve is calculated data and blue curve is measured data. (\* Colour print on P. 54)

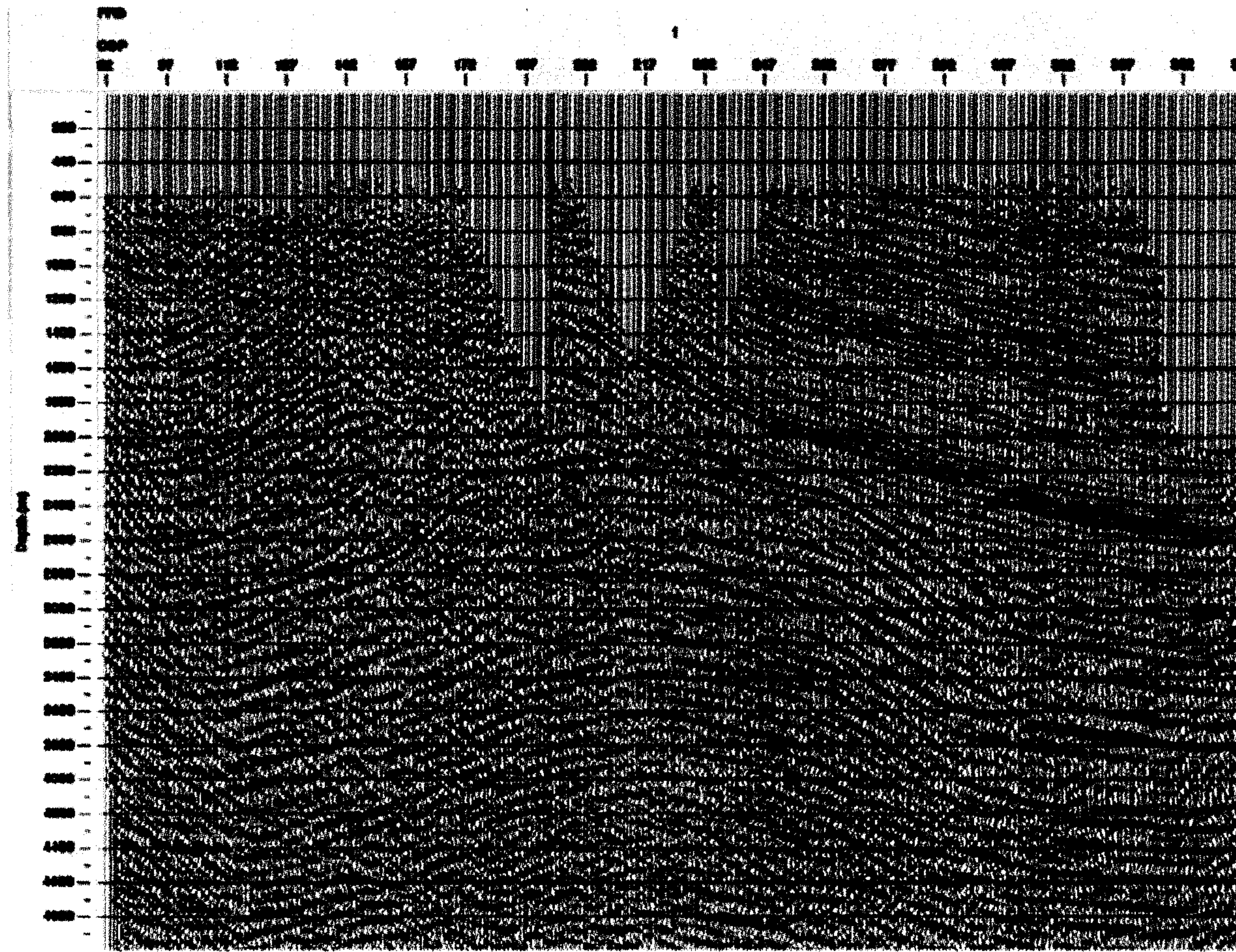


Figure 6. Depth migration of line 1131.

The inversion program has options to keep velocity gradients and/or layer thickness constant. A higher value of the damping parameter makes the algorithm produce a smoother model.

The aim of the inversion algorithm is to minimize the traveltimes misfit. This can, of course, also be achieved by not tracing rays that contribute exceptionally much to the overall rms error. An inversion effect might be that these rays do not reach the model surface again and are therefore disregarded in the misfit calculation. (Figures) Thus, a significantly decreased number of traced rays should lead to a rejection of the inversion run. A loss of traced rays always implies loss of information. Provided that the seismic phases are identified correctly, the model should only be accepted if all data or almost all data could be reproduced.

The data was then depth migrated using finite difference algorithm. The result (Figure 6) is in agreement with the geology model of the area. This algorithm was chosen after some iteration with other algorithms.

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