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Combined analytic signal and Euler method (AN-EUL) for depth estimation of gravity anomalies

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Abstract

The method expressed by Salem and Ravat. (2003) for depth estimation of magnetic sources is used on gravity anomalies for the first time.

The depth of some rectangular prisms as synthetic models are estimated through the method. The gravity effect of these models is also considered with a relatively high value of random noise. A field example is also included and the depth of the main anomaly has been estimated using this method.

Key words: Analytic signal, Euler method, AN-EUL, Depth estimation

روش ترکیبی سیگنال تحلیلی و روش اویلر برای برآورد عمق بی‌هنجاری‌های گرانی

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چکیده

در این مقاله روش معرفی شده سالم وراوات (۲۰۰۳) برای برآورد عمق بی‌هنجاری‌های مغناطیسی، برای برآورد عمق بی‌هنجاری‌های گرانی به‌کار رفته است.

عمق مکعب‌هایی با تباین چگالی‌های گوناگون ابتدا به‌منزلهٔ مدل‌های مصنوعی با استفاده از این روش برآورد شده است. به اثر گرانی این مدل‌های مصنوعی خطاهای تصادفی نیز اضافه شده است.

این روش برای مجموعه‌ای از داده‌های واقعی نیز به‌کار رفته و عمق بی‌هنجاری اصلی با این روش برآورد شده است. عمق‌های به‌دست آمده با عمق‌های تعیین‌شده از روش برآورد عمق اویلر نیز مقایسه شده‌اند و ارتباط نزدیکی بین روش پیش‌گفته و عمق‌های اویلر دیده می‌شود.

از آنجاکه عمق بی‌هنجاری‌های گرانی یکی از مهم‌ترین عامل‌های اکتشافی است تعیین هرچه دقیق‌تر عمق می‌تواند صرفه‌جویی بزرگی برای عملیات بعدی اکتشافی (حفاری) دربر داشته باشد.

واژه‌های کلیدی: سیگنال تحلیلی، روش اویلر، برآورد عمق

1 INTRODUCTION

A variety of methods, based on the use of derivatives of the gravity anomalies have been developed for the determination of source parameters such as the location of the boundaries and the depth of the bodies.

One of these techniques is the method of the analytic signal proposed by Nabighian 1972 and 1974). According to the theory of

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this method the complex signal (whose real part is given by the horizontal derivative and the imaginary part by the vertical derivative) has geometrical characteristics unambiguously related to the position and the depths of the sources. The application of the analytic signal method to the gravity anomalies was first suggested by Nabighian (1972). Klingele et al. (1991) and Marson and Klingele (1993) used the analytic signal for determining the source parameters including the depth of the gravity anomalies through gravity and gradiometeric data and by solving a non-linear equation of analytic signal of a finite step.

A rather different approach was suggested by Thompson (1982). His method is based on Euler's homogeneity equation. Solving this equation at an appropriate number of points along the gravity profile provides a system of linear equations, which in turn can be solved for estimating the position and depth of the point-source distribution.

This method found a vast application in gravity and magnetic applications (Reid et al. 1990, Klingele et al. 1991, Marson and Klingele, 1993). In this method it is not necessary to make an a prior choice of a particular geometry of the gravity perturbing bodies.

A new method combining the analytic signal and Euler method (AN-EUL) is expressed by Salem and Ravat (2003) for interpretation of magnetic sources. Their method is derived by substituting an appropriate derivative of Euler's equation into the expression of the analytic signal. Application of the method on the gravity sources including the synthetic models and the real data is the main aim of this paper.

2 THE ANALYTIC SIGNAL

The complex analytic signal in 2D based on the derivatives of gravity anomalies is expressed by Klingele et al. (1991),

$$A(x) = \frac{\partial g}{\partial x} - i\frac{\partial g}{\partial z} \qquad (1)$$

Considering equation (1) the amplitude of the analytic signal (AAS) will be (Roset et al. 1992),

$$\left|AAS(x)\right| = \sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2} \qquad (2)$$

The amplitude of the nth-order derivative analytic signal is expressed by Debeglia and Corpel (1997),

$$\left|AAS_n(x)\right| = \sqrt{\left(\frac{\partial g_n^z}{\partial x}\right)^2 + \left(\frac{\partial g_n^z}{\partial z}\right)^2} \qquad (3)$$

where subscript z denotes the vertical derivative of the field.

3 THE EULER METHOD

The 2D form of Euler equation can be defined (Kilingele et al., 1991),

$$(x - x_0)\frac{\partial g}{\partial x} + (z - z_0)\frac{\partial g}{\partial z} = -Ng(x) \qquad (4)$$

where x_0 and z_0 are the coordinates of a point source and N is the structural index.

4 THE AN-EUL METHOD

Taking the derivatives of Euler equation in the x and z directions and setting (i.e., taking the observation point above the center of the source) we get (Salem and Ravat, 2003),

$$z_0\left(\frac{\partial^2 g}{\partial z \partial x}\right)_{x=x_0, y=y_0} = (N+1)\left(\frac{\partial g}{\partial x}\right)_{x=x_0, y=y_0} \qquad (5)$$

$$z_0\left(\frac{\partial^2 g}{\partial z^2}\right)_{x=x_0, y=y_0} = (N+1)\left(\frac{\partial g}{\partial z}\right)_{x=x_0, y=y_0} \qquad (6)$$

The square root of the summation of the squares of (5) and (6) is,

$$z_0\left(\sqrt{\left(\frac{\partial^2 g}{\partial x \partial z}\right)^2 + \left(\frac{\partial^2 g}{\partial z^2}\right)^2}\right)_{x=x_0, y=y_0} = (N+1)\left(\sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial z}\right)^2}\right)_{x=x_0, y=y_0} \qquad (7)$$

This equation can be expressed as follows,

$$z_0 \left|AAS1\right|_{x=x_0, y=y_0} = (N+1)\left|AAS0\right|_{x=x_0, y=y_0}$$

where x, y are the coordinates of the measurement points and |AAS0| and |AAS1| are the amplitudes of the analytic signal of the anomaly and its first-order derivative, respectively. Equation (8) implies that the depth of the gravity source can be estimated through the AAS0 and AAS1 above the center of the source.

Therefore, in the case of a contact where N=0 we have,

$$z_0 = \left|\frac{AAS0}{AAS1}\right|_{x=x_0, y=y_0} \quad (9)$$

This equation enables us to estimate the depth of a contact above its center quite readily.

5 NUMERICCAL PROCEDURE

Synthetic gravity data for example models are computed using Talwani's algorithm (Talwani et al 1959). Random noise was added to the data. The following command was applied in MATLAB "NORMRND (MU, SIGMA., [M N])" which generates an M by N matrix of random numbers chosen from the normal distribution with parameters MU and SIGMA.

Then the horizontal and vertical derivatives of the gravity responses are computed by three point Lagrangian operator and Hilbert transform, respectively.

Having the gravity derivatives the AAS0 and AAS1 and consequently the depth over the center of the anomalies can be computed by equation (9).

6 SYNTHETIC MODELS

Data from rectangular prisms with different depths produced synthetic models.

The models and the model responses with and without noise are demonstrated in figures 1-4. The AN-EUL and Euler (EUL) methods are used for determination of the depth of the models and the results are shown in table 1.

The results demonstrate a good accuracy in the case of the narrow sources and when the maximum gravity effect is equal or more than 100 micro-Gal (Model No. 2) for the ANEUL method. In all models the results of the ANEUL are better than those of the EUL method.

7 REAL EXAMPLE

A field example is also used to test the capability of the method. The real data is gathered in an area which has been surveyed for existing Bitumen and is located close to Dehloran in the west of Iran. Limestone and dolomite are the dominant formation. The Bouguer gravity anomalies and the Euler solution of the anomalies are shown in figures 5. The gravity effect along the profile AB is demonstrated in figure 6.

The AN-EUL method is applied along the profile (AB) to estimate the depth of this negative anomaly and two depths equal to 6.7 and 15.3 meters are computed for the left and right side of the profile respectively. These depths show a good agreement with the Euler solution around the anomaly in figure 5 and excavation operations in the field.

Table 1. Depths for noisy models

Model No.	Maximum Noise (Micro Gal)	Z0(m) (EUL)	Z0 (m) (ANEUL)
1	10.7	18.5	17.8
2	3.87	11.5	9.56
3	2.45	8.2	8.5
4	2.34	16.5	17.4

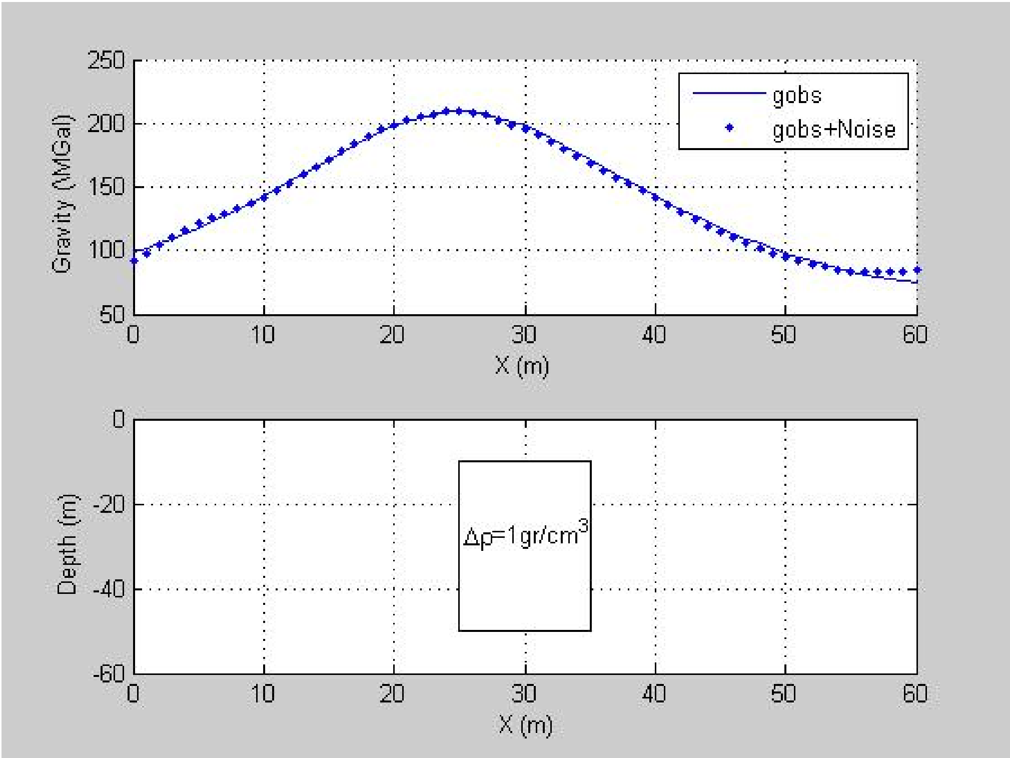

Figure 1. The gravity effects of model (10m* 40 m) in Micro-Gal.

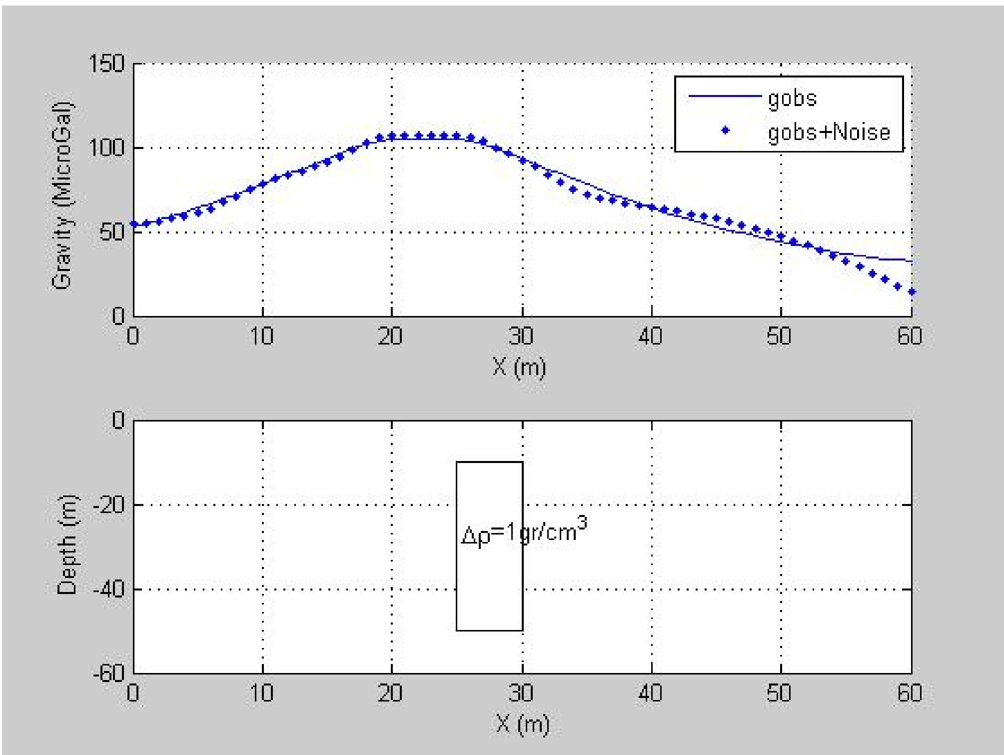

Figure 2. The gravity effects of model (5m * 40 m) in Micro-Gal.

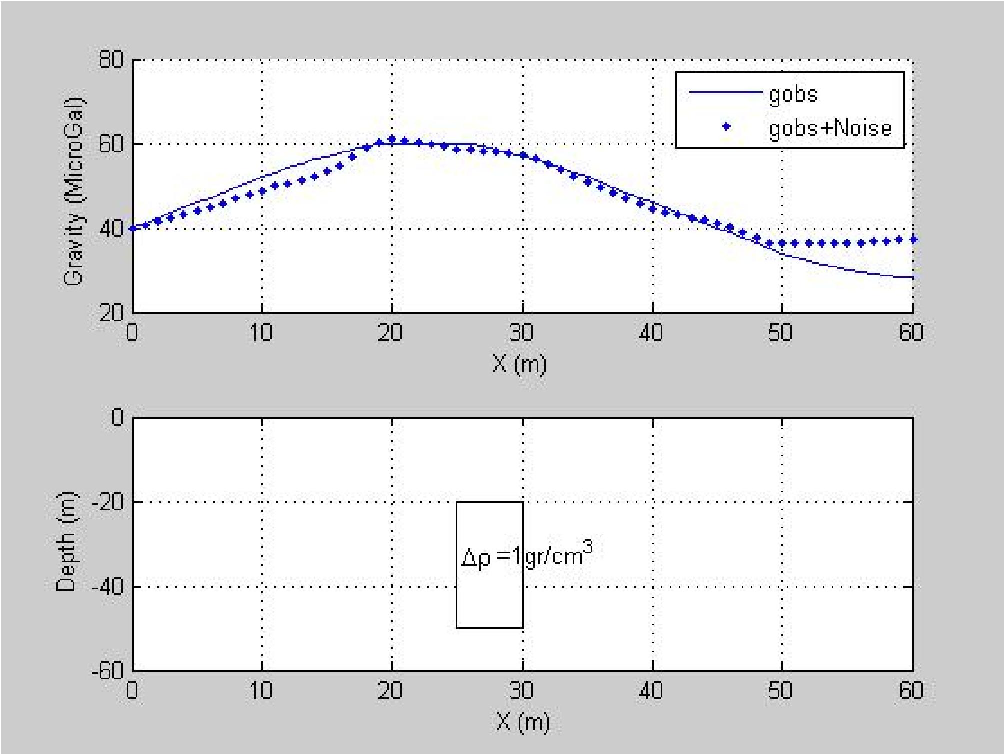

Figure 3. The gravity effects of model (5m * 30m) in Micro-Gal.

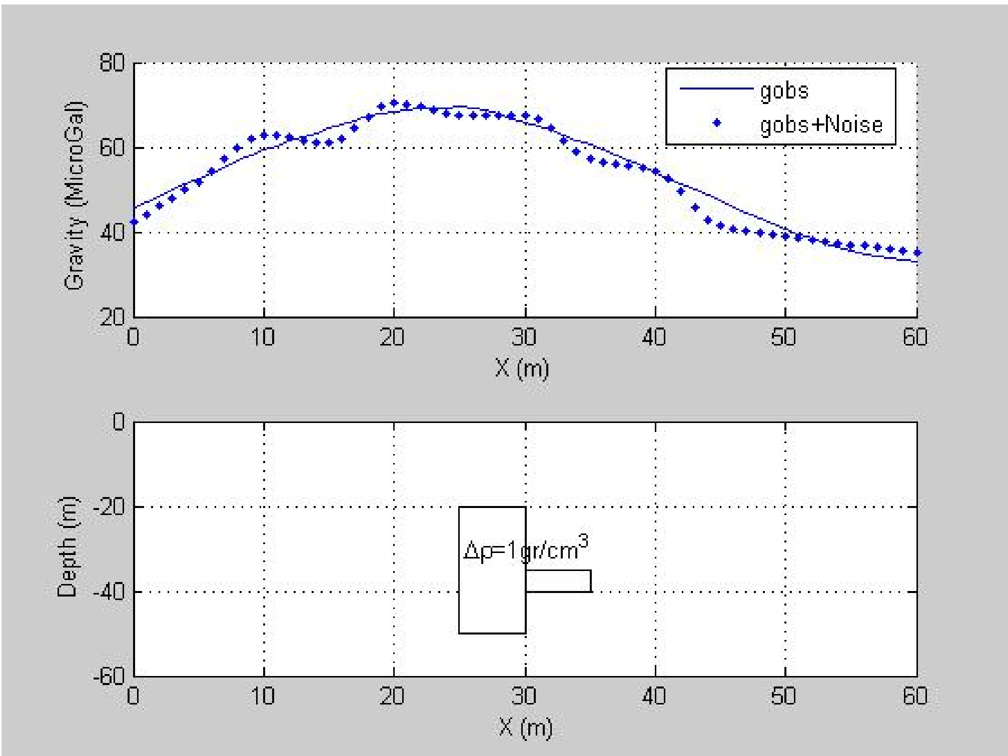

Figure 4. The gravity effects of model in Micro-Gal.

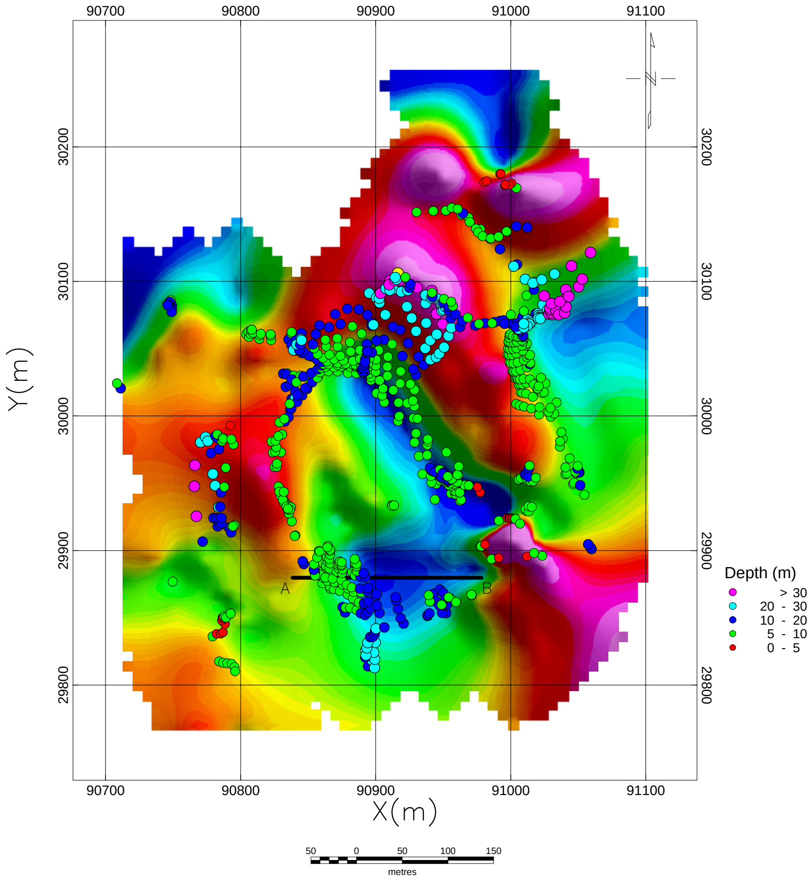

Figure 5. The Bouguer gravity anomalies (mGal) and Euler depths.

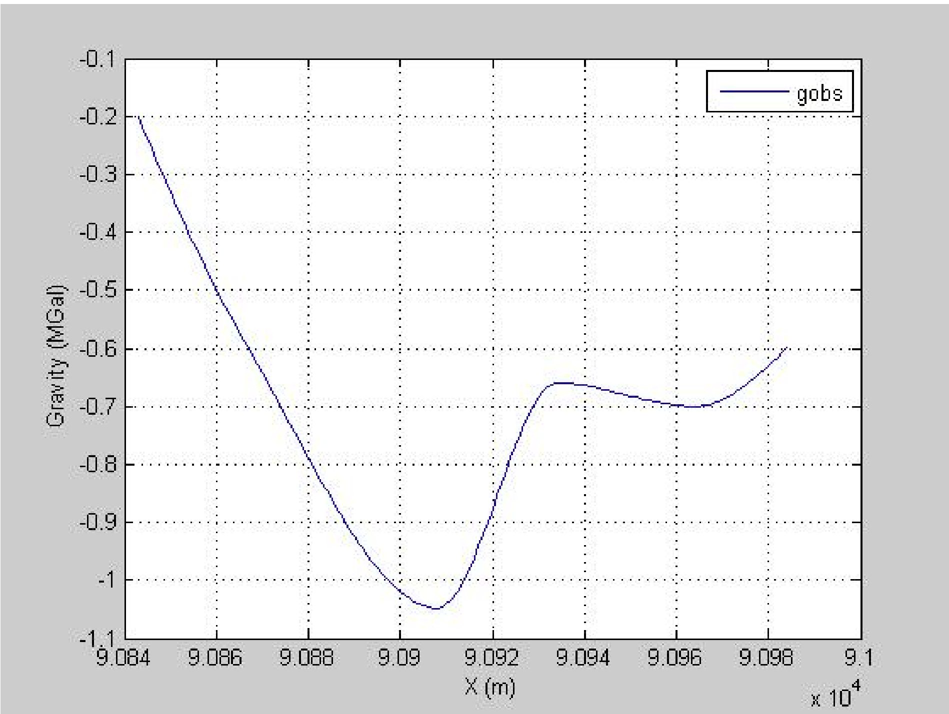

Figure 6. The Bouguer gravity anomalies along profile AB.

8 CONCLUSION

The method which has originally been defined for determination of the depths of the magnetic anomalies is quite capable in the case of the gravity anomalies.

The method is quite feasible and is applicable with a few numerical computations and estimates the depth of the top of the narrow anomalies such as dikes and in the presence of random noises.

However, when the maximum gravity effect of the source is less than 100 Micro-Gal or the width of the source increases (say equal or greater than 20 percent of the length) the accuracy in the depth of the top of the anomaly decreases substantially.

In the case of complex sources (Model No.4) the depth of the larger part of the source is quite dominant.

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