# Depth estimation of gravity anomalies using Hopfield Neural Networks

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**Abstract**

The method of Artificial Neural Network is used as a suitable tool for intelligent interpretation of gravity data in this paper.

We have designed a Hopfield Neural Network to estimate the gravity source depth. The designed network was tested by both synthetic and real data. As real data, this Artificial Neural Network was used to estimate the depth of a Qanat (an underground channel) located at north entrance of the Institute of Geophysics and the result was very near to the real value of the depth.

**Key words:** Artificial Neural Network, Gravity, Depth estimation, Hopfield

## تخمین عمق بی هنجاریهای گرانی با استفاده از شبکه های عصبی هاپفیلد

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**چکیده**

در این مقاله روش شبکه عصبی هاپفیلد برای تفسیر هوشمند داده های گرانی استفاده شده است. یک شبکه عصبی هاپفیلد برای تخمین عمق چشمه گرانی طراحی شده است. این شبکه طراحی شده برای داده های مصنوعی و واقعی آزمایش شده اند.

در مورد داده های واقعی این شبکه برای تخمین عمق یک تونل قنات واقع در موسسه ژئوفیزیک به کار برده شده و نتایج حاصله به مقادیر واقعی عمق بسیار نزدیک است.

**واژه‌های کلیدی:** شبکه عصبی، گرانی، تخمین عمق، هاپفیلد

## 1 INTRODUCTION

Neural Networks are being increasingly used in prediction, estimation, and optimization problems. Neural networks have gained popularity in geophysics in the last decade (Gret and Klingele, 2000).

They have been applied successfully to a variety of problems in geophysics. Nowadays Neural Networks can be applied in microchip technology for computer hardware.

Neural networks have been applied in interpreting well logs (Huang et al.,1996), recognizing seismic waveforms (Ashida, 1996) and automatic detection of buried utilities and solid objects from GPR data (Al-Nuaimy et al.,2000). In addition, applications to the interpretation of magnetic data have been reported. In these applications, the back propagation network was used for structural interpretation of aeromagnetic data (Pearson et al.,1990), classification of buried objects from their magnetic signatures (Brown, et al.,1995) and more recently detection of tunnels from gravity data (Salem et al.2001).

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Recent developments in gravity measurements and especially in microgravity tools have been prepaid an excellence condition of data acquisition to have better interpretation results depth estimation of gravity sources.

These developments, combined with higher speed data acquisition technology, have made it possible to detect much smaller objects like small cavities, chromites lenses, etc.

The gravity data sets are naturally noisy so that it is very hard to estimate the gravity source depths precisely.Therefore, there is an increasingly need for a fully automatic interpretation that can be used to make decisions regarding the nature of the source in near real time. The massively parallel processing advantage of Artificial Neural Networks makes them suitable for hardware implementation; therefore, detection of gravity sources objects will be possible more precise.

The interpretation of gravity data depends greatly on the experience of the interpreter and there is an increasing need for intelligent interpretation techniques that can be used to make rapid decisions. Especially it is very important to especially mention that some techniques like the Euler method, Analytical signal, Up-ward Continuation and Down-ward Continuation are very sensitive to noise and can estimate only the edges or around the gravity source location.Therefore, a technique not very sensitive to noise and more precise is needed for depth estimation of gravity anomalies.

One such technique may be found in the emerging field of Artificial Neural Networks.

In this paper, we have developed a new method for detection and depth estimation of gravity anomalies using the Hopfield Neural Network applied to gravity data. The Hopfield networks have already been electrically and electro-optically implemented in a package as a chip which can be used in future generations of gravimeters instrumentations.

## 2 ARTIFICIAL NEURAL NETWORKS

Artificial neural networks are part of a much wider field called artificial intelligence, which can be defined as the study of material facilities through the use of computational models (Charniak and McDermott.1985).They encompass computer algorithms that solve several types of problems. The problems include classification, parameter estimation, parameter prediction, pattern recognition, completion, association, filtering, and optimization (Brown and Poulton, 1996).

There are several types of artificial neural networks. For complete information covering the whole domain of neural networks kinds, the reader is referred to the excellence book of Fundamentals of Artificial Neural Networks by Menhaj (2000).

Summarizing their reviews, neural networks can be divided into two main categories: supervised feed-forward networks and unsupervised recurrent networks. In the supervised feed-forward, information is only allowed to flow in one direction without any feedback. These nets are supervised because using a set of correct input-output pairs, called the training set; small changes in the connection weight are made in order to minimize the difference between the actual and the desired output values in a distributed way. Back propagation is the most popular supervised feed forward network. In the supervised recurrent type the network allows information to flow in either direction. These models are called unsupervised and useful in optimization applications where a certain cost function should be minimized. One should merely choose a neural network whose energy function coincides with the given cost function. A Hopfield model is the most popular unsupervised recurrent network. Farhat and Bai (1989) and Kulkarni (1991) have used the Hopfield network to solve some general ill-posed problems. In geophysical applications ,Wang and Mendel (1992) employed it in seismic deconvolution while Zhang and Paulson (1997) used this net to invert magnetotelluric data.

An application of neural network in

gravity is in its early stages. In the future we hope to develop a more flexible intelligent method for gravity interpretations with applying other methods in addition neural networks like Fuzzy Logic, Genetic Algorithms, specially to develop a near-real-time processing system.

In this paper, we used the unsupervised recurrent Hopfield neural network.

## 3   HOPFIELD NETWORK

The Hopfield model is a single layer feedback neural network. It means that the flow of data is not only from one direction and this network incorporates feedback into the neuron from all neurons except itself.

On the other hand, in this type of neural network the neurons are all connected to each other and the weights of the forward connections are the same as their reverse connection, so the data entered in the network have the effects on all of the neurons available. The weight of the neuron connections is fixed and can be calculated by Hopfield methodology. As it has been shown in the figure (FIG.1)it is a 4-neuron Hopfield network diagram. The sum square error of the Hopfield network output is the energy function which must be minimized to get the best results.

So in the Hopfield network we have an energy function which may be defined as:

$$E(v)= - \frac{1}{2} \sum_{i=1}^{n} \sum_{j \neq i=1}^{n} WijViVj - \sum_{i=1}^{n} IiVi$$

(1)

Where:
E(v):The energy function of Hopfield network
n: total number of neurons
Wij: The weight of connection from neuron i to neuron j
Vi:the i element of input vector V
Vj: the j element of input vector V
Ii:the I'th input also known as threshold

It has been shown that if weight Wij is the symmetric matrix, the energy function  E(v) will never increase as the state of the neurons(V1,V2,…,Vn) change (Hopfield , 1984; Hopfield and Tank, 1985; Tank and Hopfield,1986). This means that the network will converge to a state at which the energy function E(v) is locally minimized (Wang and Mendel, 1992). Accordingly, the Hopfield network can invert any set of measured geophysical data to another set of model parameters if a cost function can be formulated between the measured data and those theoretically calculated based on the model parameters The Hopfield network can be used in geophysical applications analogous to the conventional inversion techniques such as least squares method. However, inversion of geophysical data in general is not a simple task. The main fundamental difficulties are the problem of no uniqueness and instability in the solutions (Li and Oldenberg, 1996).In this work, the Hopfield network is not charged to do a classical inversion of gravity data. We have taken a simple approach but it may be convenient to real time detection.
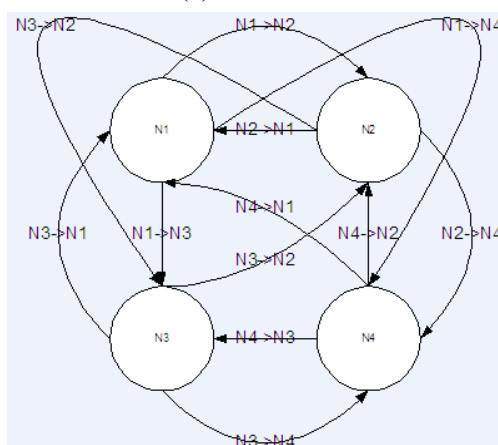


**Figure 1.** A 4-neuron Hopfield Neural Network diagram.

The Hopfield neural network has proven to be powerful for solving a wide variety of optimization problems (Hopfield and Tank, 1985, Tank and Hopfierls, 1986).The key step in applying the Hopfield neural network to an optimization problem is to relate a suitable cost function of the optimization problem to the Hopfield energy function. Once this relationship is formulated, the network changes from its initial state to final state. The final state constitutes the solution of the problem where the energy function is minimized.

In this work, we follow the error function as the energy cost function which should be minimized.

## 4   DEPTH ESTIMATION OF GRAVITY ANOMALIES

The general gravity effect at an observation point(x,z=0)caused by simple body models like sphere and horizontal cylinder at x=0 and buried at a depth of z(Fig.2) is given by equation 2
(Abdelrahman et al., 2001)
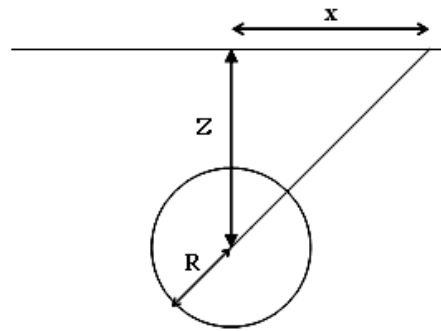
$$g = \frac{A}{(x^2 + z^2)^q} \qquad (2)$$



**Figure 2.** Geometrical specifications of a simple body model.

Where q is a value characterizing the nature of the object:
a)If the object is a horizontal cylinder mode l: q=1
b) If the object is a spherical model: q=1.5
And A is the amplitude factor

It is a good estimation for some objects near to spherical or cylindrical shape that the value of q is in the range of 1 to 1.5. It is clear from equation(2), that the horizontal location of the object can be estimated from the Bouger anomalies contours and residual anomalies contours. For example as shown in figure (Fig.3) it is very clear from the intensity of the colors of the Bouger contours where the horizontal location of the object is.
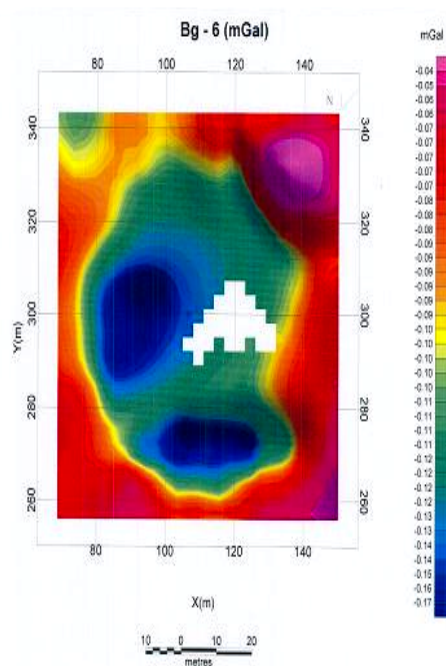


**Figure 3.** Bouger anomaly map.

Suppose we have M gravity data measured over an object of unknown amplitude factor A and which is located at the position $(\ell)$ .To estimate the unknown amplitude factor, we build a cost function between the measured and calculated gravity anomaly of the model .The theoretical gravity anomaly at a measured point k can be written:

$$g_k^c = G_{lk} A \qquad (3)$$

Where A is the amplitude factor and $G_{lk}$ represents the geometrical relation between the position $(\ell)$ and the observation point k.

There is a problem here that if we have the noisy data, how can the amplitude factor be calculated? This can be approximated in a least square method by a solution that minimizes a cost function between the measured and calculated data. We define a cost function C in terms of the some squares of the differences between the measured and calculated data (Salem,et al, 2001):

$$C = \frac{1}{2}\sum_{k=1}^{M}(g_k - g_k^c)^2 = \frac{1}{2}\sum_{k=1}^{M}[g_k - G_{lk}A]^2$$

$$(4)$$

Where $g_k$ represents the measured gravity data. The amplitude factor A must be represented in a system consistent with the output of the Hopfield network, where a typical bit can be 1 or 0.Referring to binary digits rules the amplitude factor can be expressed as:

$$A = \sum_{i=1}^{n=D+U+1} 2^{i-D-1}b_i \qquad (5)$$

Where D is the number of down bits of the binary value and U is the number of its up bits and b is a binary digit (0 or 1). It is clear that D and U depend on the precision and amplitude, respectively.

Substituting equation (5) into equation (4), expanding, and regrouping gives the connection weight and input initial values as mentioned below.

$$W_{lij} = -\sum_{k=1}^{M} 2^{(i+j-2D-2)}(G_{lk})^2 \qquad (6)$$

$$I_{ij} = \frac{1}{2}\sum_{k=1}^{M}(2^{(i+j-2D-2)}G_{LK})^2 b_i + \sum_{k=1}^{M}2^{(i-D-1)}G_{lk}g_k$$

$$(7)$$

So the Hopfield energy to estimate the amplitude factor at location $(\ell)$ becomes:

$$E_l(b) = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i=1}^{n}W_{ij}b_j b_i - \sum_{i=1}^{n}I_{li}b_i \qquad (8)$$

We selected 9 neurons for the Hopfield neural network because the accuracy of gravity data was $1\,\mu$ Gal and so 9 bits are needed to present the amplitude factor of gravity value in binary digits.(As mentioned before the binary value of the amplitude factor of gravity data consists of D+U+1 digits, refer to equation (5).

We applied different values of depth (Z) and calculated for each of them the amplitude factor by the Hopfield network and the final minimized cost function of that Hopfield value was obtained for them.

The depth value in which the cost function has the minimum value is the nearest value to the real depth of the gravity source.

## 5    SYNTHETIC DATA AND THE HOPFIELD NETWORK ESTIMATION IN PRACTICAL CASES

After calculating the weights and biasing values of the network it rotates while getting to a stable condition, it means that after calculating the weights and biasing values they will be fixed but the initial value of depth will be applied to the rotating network until a condition is reached where its output is the same as the previous output; in this state it is said that the energy of the neural network has reached its minimum. It is similar to a ball which runs along a topographic area with valise and tops then it will be stopped in one of the valise which has the minimum depth. So we prepared a Hopfield neural network program that tested it for different initial values of depth and then this program calculates the energy of the network in its stable state. The output of the network which has the least energy means the

depth estimation which is near to the actual depth of the source.

To test the behavior of the network we fed the noisy data of a cylinder where the ratio of signal to noise was 5%, 10%.After the network became stable, the depth was estimated by the program mentioned before. The results for this test have been presented in Table 1.

As the results show, the network has a good ability to detect the near surface object in the presence of other noises like heavy buildings and tunnels or cavities which may be affecting the object near to it. This is a very hard task to distinguish the coefficients of a suitable filter to attenuate the effects of the noise on the main signal but fortunately

the artificial neural network is less sensitive to noise.

As it has been shown in Table1, the network was tested for synthetic gravity data of cylinder and sphere models in the presence of noise and the values of depth(Z)and amplitude factor (A) were caculated.It is clear from the table that depth estimation in the presence of 10% noise is near to its training value.

As real data we tested the network for the gravity data measured on the surface of the ground on top of a subterranean canal (Qanat) at the north entrance of the Institute of Geophysics. We first selected a principle profile data from the gravity network profiles (Fig4).

**Table1.** Outputs of Hopfield Neural Network in presence of 10% noise.

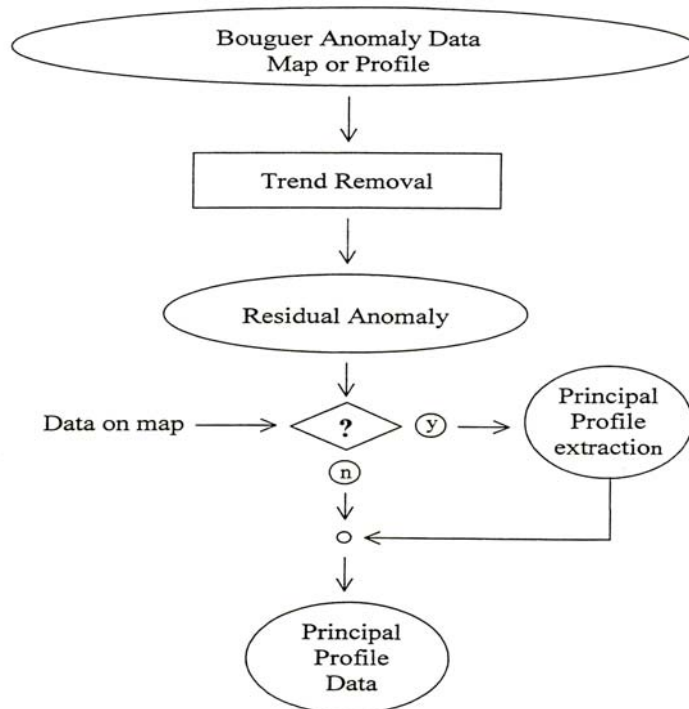| Training values for R,Z | | outputs of MLP(3,5,2) in presence of 10% noise | | | |
|---|---|---|---|---|---|
| Horizontal    cylinder | Sphere  or Vertical cylinder | Horizontal cylinder | | Sphere  or Vertical cylinder | |
| R(m) | Z(m) | R(m) | Z(m) | R(m) | Z(m) |
| 1 | 2 | 1.17 | 2.25 | 1.12 | 2.22 |
| 1 | 3 | 1.22 | 3.46 | 1.08 | 3.45 |
| 2 | 4 | 2.15 | 4.23 | 2.09 | 4.25 |
| 2 | 5 | 2.18 | 5.33 | 2.14 | 4.28 |
| 3 | 6 | 3.25 | 6.45 | 3.17 | 6.53 |
| 4 | 8 | 4.17 | 8.43 | 4.28 | 8.76 |
| 5 | 13 | 5.65 | 13..85 | 5.30 | 13.45 |
| 6 | 14 | 6.25 | 14.18 | 6.31 | 13.40 |
| 6 | 15 | 6.25 | 16 | 6.35 | 14.65 |

**Figure 4.** Flowchart of selecting principle profile data.

The principal profile data is a profile which is perpendicular to the extension of the anomaly. So we selected the profiles shown in figure 5(lines A and B).

In the next stage the simple body of sphere or Cylinder was assumed and the values of G and gk (in equations 6,7) were extracted from the principle profile data. Then the weights Wij and thresholds Iij are calculated via equations 6&7 to minimize the energy function. Then different random initial values of amplitude factor (A in equation 2) will apply to the Hopfield network .
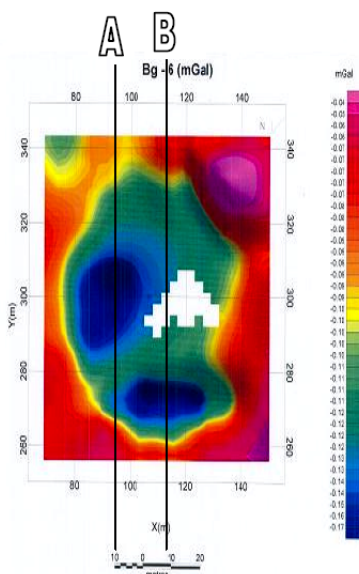


**Figure 5.** The principle profiles selected for the real data.

As the accuracy of gravity data was $1\mu$ Gal ;9 bits are needed to present the amplitude factor of gravity value in binary digits So we used the 9-neuron Hopfield neural network .The network turned until the new output was equal to the last stage. In this condition the network reaches its stable state that means the energy function is minimized. The energy function for each initial value is calculated using equation 1 and the output of the network which has the minimum energy function is the real amplitude factor. Following the mentioned stages above the result from the Hopfield neural network was 2.5 meter for profile A and 9.5 meter for profile B, compared to the real depths which were respectively  2.9 meter and 9.25 meter; this method presented good accuracy of depth estimation.

## 6   CONCLUSIONS

It this paper a new method has been proposed for intelligent interpretation of gravity data for exploration, especially in depth estimations. The observed gravity anomaly of the object is assumed to be produced by an equivalent source of cylinder or sphere, which has an amplitude factor related to the radius, density contrast and depth. We used the Hopfield neural network to optimize the amplitude factor of the source at a set of subsurface targets. For each target location the network was run and its stable energy was calculated and after that the one which has the minimum energy in its stable state was supposed as the nearest value to actual location and depth. Also we tested the network for synthetic data of the two models of sphere and cylinder in the presence of noise and saw the results have good adaption to actual values. For a testing of field data we measured the gravity points of the ground on top of a subterranean canal and fed the corrected data, after pre-filtering, to the network .The depth estimation by the network (2.5 meter) was very near to the real value of the depth (2.9meter).

In the future we will try to develop this method for higher noisy signals especially in a fuzzy logic methodology joined to genetic algorithm to have access for other complicated shapes of objects.

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