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Gravity acceleration at the sea surface derived from satellite altimetry data using harmonic splines

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Abstract

Gravity acceleration data have grand pursuit for marine applications. Due to environmental effects, marine gravity observations always hold a high noise level. In this paper, we propose an approach to produce marine gravity data using satellite altimetry, high-resolution geopotential models and harmonic splines. On the one hand, harmonic spline functions have great capability for local gravity field modeling. On the other hand, the information from satellite altimetry is a viable source of information for the marine gravimetry in the high-frequency gravity field modeling. Marine geoid from satellite altimetry observations can be converted to disturbing potential via ellipsoidal Bruns's formula. The reference gravity field's contribution is removed and restored after solving Dirichlet Boundary Value Problem. Finally, the results are downward continued to the sea surface using free air scheme. Computation of gravity acceleration in the Persian Gulf and its compatibility with the shipborne data shows reasonable performance of this methodology.

Keywords: Harmonic splines, Shipborne gravimetry, Satellite altimetry, Gravity field modeling

1 Introduction

The Earth's gravity field modeling in marine regions for geoid determination, prospecting and exploration with high accuracy is the main goal among researchers in geodesy and geophysics community. Due to environmental disturbance and fluctuations in the ship movement, shipborne gravimetry observations are usually highly noisy. Moreover, because of the vast area of the oceans and water bodies and slow rate of data collection due to low velocities of ships, it is nearly impossible to have concurrent measurements. It is also economically impossible mission to provide a homogeneous global coverage of marine data.

Satellite altimetry has provided a new source of information for marine geoid determination over the sea areas. It should be noted that satellite altimetry provides accurate measurements on the order of centimeter, reducing to the order of decimeter in coastal areas (Anzenhofer et al., 1999). Such accuracy in the geometric space is equivalent to the order of microgal in the gravity space (Safari et al., 2005). Therefore, one can see altimetry data as relatively accurate source of information for gravity field applications.

The geodetic community has widely studied the gravity field determination via satellite altimetry data. The interested reader can find valuable contributions from Andersen and Knudsen (1998), Hwang (1998), Tzivos and Forsberg (1998), Hwang et al. (1998), Andersen and Knudsen (2000), Hwang et al. (2002), and Sandwell and Smith (2009). Nearly all of the above researchers

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have employed Stokes, Hotine or Vening Meinesz integrals to produce gravity field functionals from the geoid undulations measured by satellite altimetry.

In this paper, we introduce a method for determine gravity acceleration at the sea surface based on satellite altimetry observations and harmonic splines. In contrast to previous methods, geoid undulation from satellite altimetry data is used for computation of disturbing potential via ellipsoidal Bruns's formula (Ardalan and Grafarend, 2001; Safari et al, 2005). Disturbing potential is the difference between actual gravity potential of the real Earth and that of the normal gravity potential at the evaluation point (Ardalan and Grafarend, 2004). With knowledge of geoid potential, W_0 (Bursa et al, 2007), which is equal to normal potential at the surface of the reference ellipsoid (Safari, 2012), disturbing potential is used to compute the actual gravity potential of the Earth at the surface of the reference ellipsoid. Actual potential at the surface of the reference ellipsoid can be divided into two parts: (1) reference part, i.e., effect of the reference gravity field; and (2) residual potential. In order to achieve residual potential, one can remove the effect of the reference gravity field of the actual potential. The reference gravity field consists of three parts: a) the modeled gravitational field, from ellipsoidal harmonics expansion of the external gravitational field up to degree and order 240, b) ellipsoidal centrifugal field, and c) the effect of sea masses outside the reference ellipsoid surface.

Residual potential satisfies the Laplace differential equation in the outer space of the reference ellipsoid; thus holding only for harmonic quantities. In order to solve the Dirichlet Boundary Value Problem (BVP) (main step in applied method to produce gravity acceleration), harmonic splines interpolation described by Freeden (1987) is applied. For further details we refer to Freeden (1981, 1987, and 1990) and Freeden and Michel (2004).

After solving Dirichlet BVP, a specific solution to the Laplace differential equation in the ellipsoidal coordinate system, we can apply any linear operator to express other gravity quantities such as gravity acceleration.

The main steps of the proposed method are shown algorithmically in Figure 1.

This paper is organized as follows: Section 2 describes discrete exterior Dirichlet problem and its solution based on harmonic splines. Application of the harmonic splines for production of gravity acceleration at the sea surface is presented in Section 3, followed by numerical evaluation of the applied technique at the Persian Gulf. Conclusions will be presented in Section 4.

2 Discrete Exterior Dirichlet problem for the residual potential

After removing the effect of reference gravity field from actual potential, we obtain residual potential δW at the surface of the reference ellipsoid $\sum$, boundary of problem as a smoothed regular surface. The residual potential satisfies the Laplace equation in the outer space of the reference ellipsoid.

The traditional approach for the gravity field modeling is to use the spherical harmonics as the base functions. The most significant weakness of the traditional method is that the harmonics have a global support and cannot be localized in the space domain, while these functions have ideal localization properties in frequency domain (Sneeuw, 2006). For local gravity modeling, we need spaces with base functions having ideal localization properties in space and frequency domains. According to the uncertainty principle, however, the ideal localization in both space and frequency domains is not possible. Increasing the localization in the space domain decreases the localization in the frequency domain and vice versa. This problem can be solved using a group of spherical kernels. These kernels have high capabilities in the high-frequency gravity field modeling. In recent years, spherical splines and spherical wavelets have been of great interest in the local gravity field modeling (Freeden and Michel, 2004).

Having an ideal localization in the space domain, harmonic spline interpolation (Freeden, 1987) can be used to solve the Dirichlet BVP. The data points $\left(x_i, F(x_i)\right) \in \sum \times \mathbb{R},\ i = 1, \dots, N$ correspond to a set of discrete points on $\sum$ (Freeden and Michel, 2004). Residual potential $\delta W_N^F(x)$ is estimated in the external space of the reference ellipsoid as follows (Freeden, 1987):

$$\delta W_N^F(x) = \sum_{i=1}^{N} K_H(x_i, x) a_i \ , \ \ x \in \overline{\sum_{ext}} \tag{1}$$

where the unique coefficients $a_1, \dots, a_N$ satisfy the linear system of observation equations in the following form:

$$\sum_{i=1}^{N} K_H(x_i, x_j) a_i = F(x_j) \ , \qquad j = 1, \dots, N \tag{2}$$

where $K_H(x_i, x_j)$ is reproducing kernel in Hilbert space (H) and linearly independent functions $K_H(x_1, .)$,…, $K_H(x_N, .)$ are called Harmonic Splines in H relative to system $\{x_1, \dots, x_N\}$. (Freeden and Michel, 2004). Table 1 shows some examples of such kernels. Due to high capability of the Poisson's kernel in the spatial localization (Glockner, 2002), we have applied this kernel. $\delta W_N^F(x)$ is an approximation of the residual potential δW at points over and outside the surface of the reference ellipsoid, and it is a member of the function space of the regular harmonic functions outside the Bjerhammer sphere with radius α (Klees et. al., 2008).

If the quantities $F(x_1), \dots, F(x_N)$ are affected with errors, the interpolation should be replaced by the smoothing (Freeden, 1981; Freeden, 1999; Moritz, 1980; Wahba, 1990). The coefficients $\boldsymbol{a}$ are uniquely determined by the following linear system:

$$(K_H + \lambda \mathrm{I})\boldsymbol{a} = \boldsymbol{F} \ , \qquad \boldsymbol{F} = (F_1, \dots, F_N) \tag{3}$$

where λ is a positive constant and is the optimal smoothing parameter to convert the interpolated splines into smoothing splines (Freeden, 1981 ; Freeden, 1987 ; Freeden, 1999). K_H is positive definite, hence, $K_H + \lambda \mathrm{I}$ is positive definite too, and the above system is uniquely solvable.

3 A case study: validity control of gravity acceleration in the Persian Gulf

In this section, we present an application of the method to produce gravity acceleration in the Persian Gulf (the study area: $47 \le \lambda \le 57, 23 \le \varphi \le 31$). This method has not been used previously in Iran. Figure 2 illustrates the plot of mean sea level (MSL) variations computed based on CSRMSS95 satellite altimetry model (Kim et al., 1995) over the test area. The POCM-4B model has been used here to calculate the Sea Surface Topography (SST). This model has been verified from daily observations of the wind stress field and monthly observations of the mean sea surface heat fluxes from 1987 to 1994 (Stammer et al., 1996), and is provided in terms of the coefficients complete to degree 360 of spherical harmonics (Rapp, 1998). The SST variations at the study area using the data from this model are displayed in figure 3.

Table 1. Analytical expressions for some reproducing kernels (Freeden and Michel, 2004).

Abel-Poisson	$K_H(x,y) = \frac{1}{4\pi} \frac{\lvert x\rvert^2 \lvert y\rvert^2 - \alpha^4}{(\lvert x\rvert^2 \lvert y\rvert^2 - 2(x.y)\alpha^2 + \alpha^4)^{\frac{3}{2}}}$
Singularity	$K_H(x,y) = \frac{1}{2\pi} \frac{1}{(\lvert x\rvert^2 \lvert y\rvert^2 - 2(x.y)\alpha^2 + \alpha^4)^{\frac{1}{2}}}$
Logarithmic	$K_H(x,y) = \frac{1}{4\pi\alpha^2} \ln\left(1 + \frac{2\alpha^2}{(\lvert x\rvert^2 \lvert y\rvert^2 - 2(x.y)\alpha^2 + \alpha^4)^{\frac{1}{2}} + \lvert x\rvert \lvert y\rvert - \alpha^2}\right)$

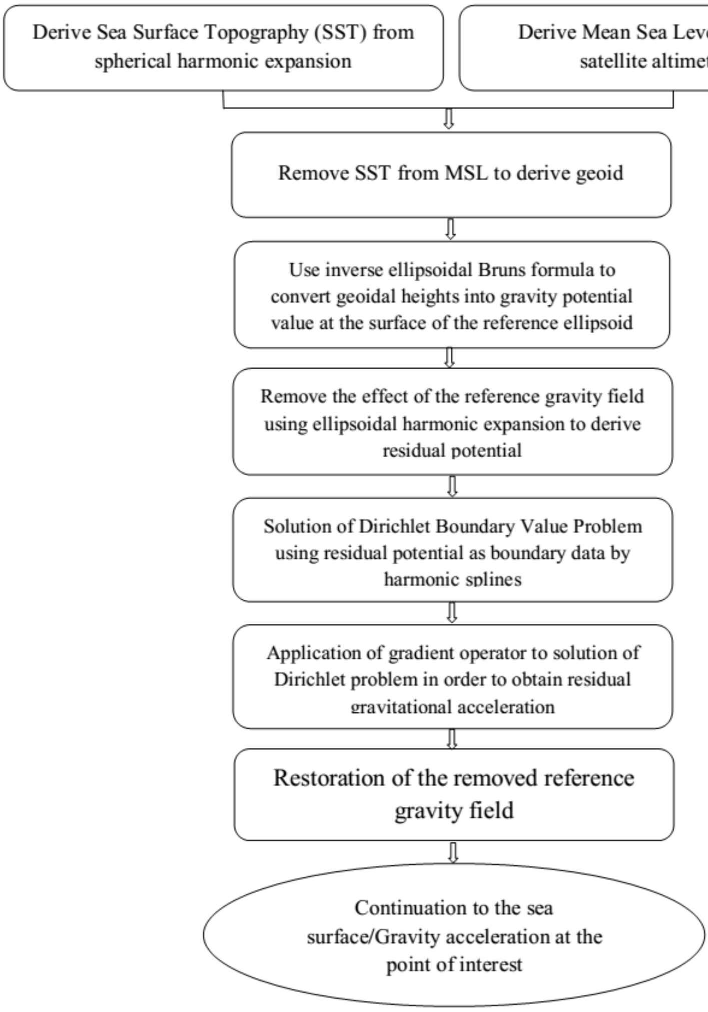

Figure 1. Flowchart of the proposed method.

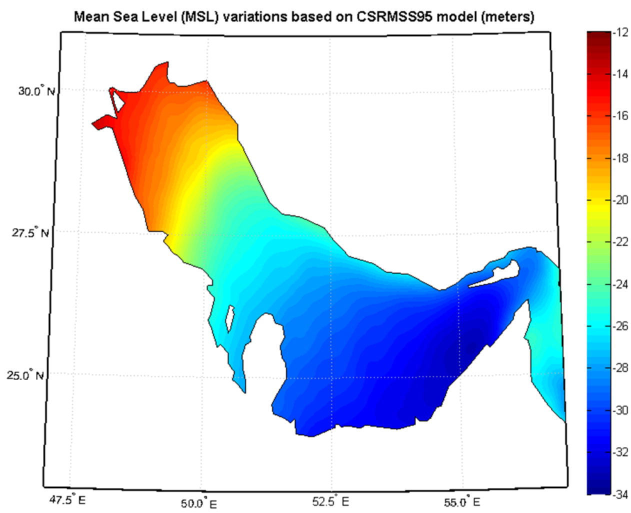

Figure 2. Mean sea level spatial variations (in meters) over the Persian Gulf based on satellite-altimetry observations.

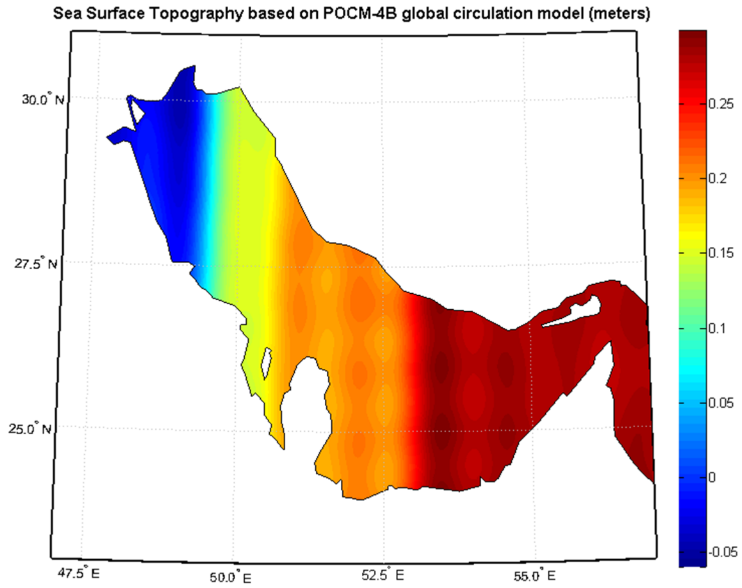

Figure 3. Spatial variations of sea surface topography over the Persian Gulf (in meters).

Figurer 4 shows variations of the geoid height from the reference ellipsoid in the Somigliana-Pizzetti field (WGD2000 ellipsoid) over the region using satellite-altimetry data.

The marine geoid computed based on satellite altimetry data converted to disturbing potential via the ellipsoidal Bruns formula. Variations of disturbing potential at the surface of the reference ellipsoid over the Persian Gulf are plotted in Figure 5.

Actual potential W at the surface of the reference ellipsoid is obtained by adding geoid potential W_0 to disturbing potential from the ellipsoidal Bruns formula (Bursa et al., 2007). Figure 6 shows the variations in the true gravity potential values over the Persian Gulf.

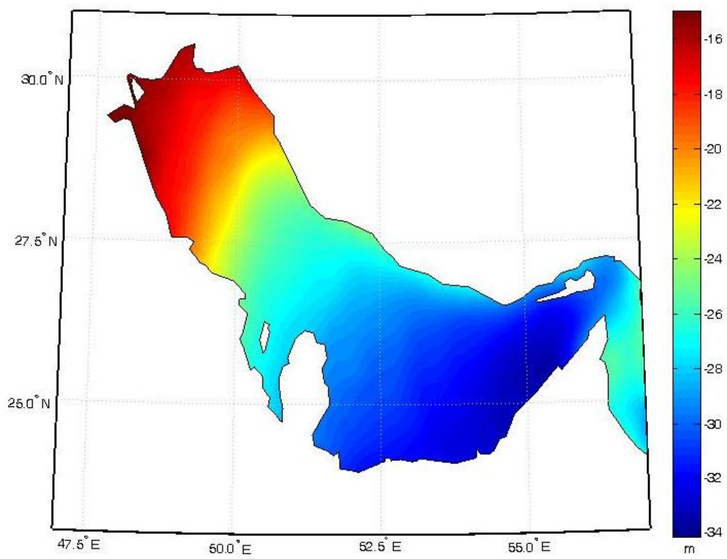

Figure 4. Geoid variations over the Persian Gulf (in meters).

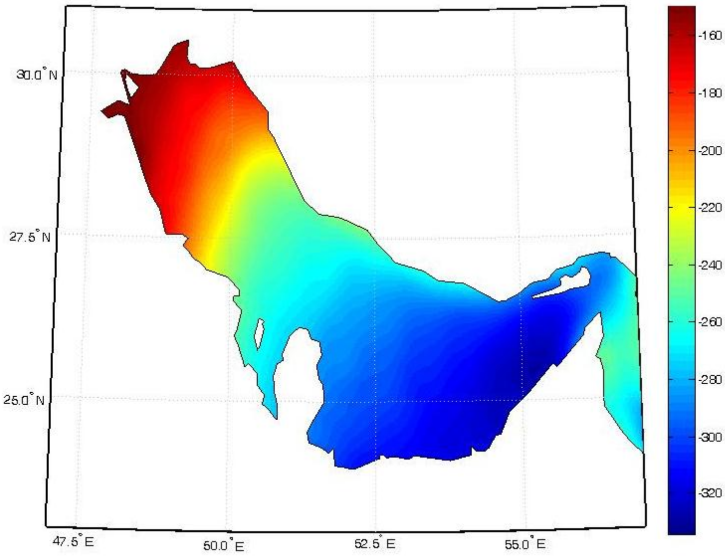

Figure 5. Disturbing potential over the Persian Gulf (m^2/s^2).

In figure 8, we plotted variations of the residual potential after removal of the effect of the reference gravity field from the actual potential at the surface of the reference ellipsoid in the test area (Figure 7). The reference gravity field is a model presented by an ellipsoidal harmonic expansion of gravitational potential up to degree/order 240/240 plus the ellipsoidal centrifugal field. At the remove and restore steps, the EIGEN-GL04C geopotential model (Forste et al., 2005) was used as the reference gravitational field. The spherical harmonic coefficients of the EIGEN-GL04C model were transformed into the ellipsoidal harmonic coefficients using the exact transformation relation of Jekeli (1988).

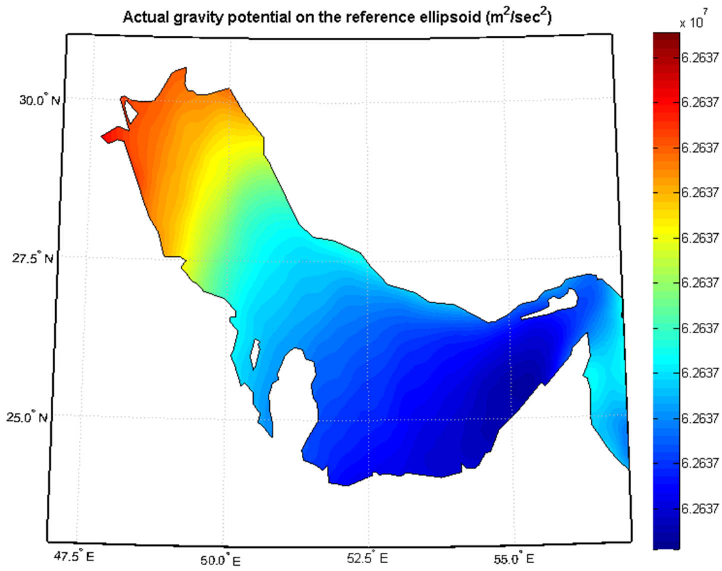

Figure 6. Variations in true gravity potential at the surface of the reference ellipsoid (m^2/s^2).

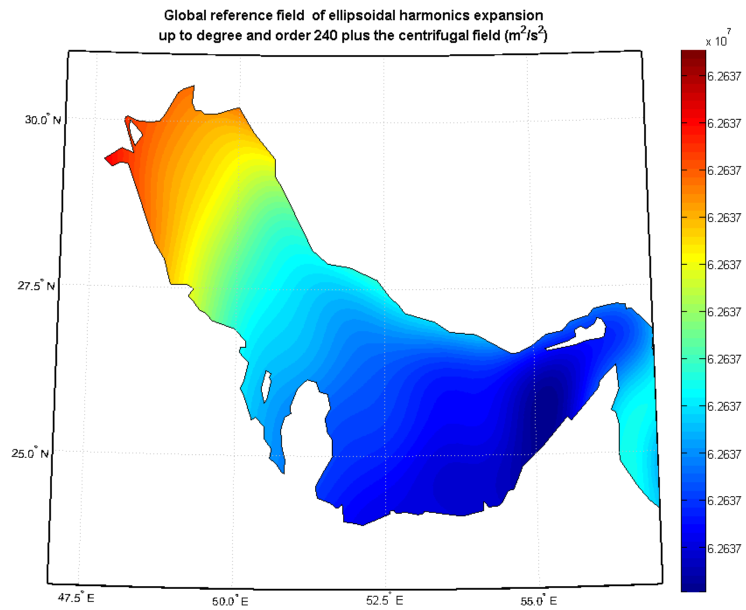

Figure 7. The effect of the reference field as the ellipsoidal harmonic series expansion to the degree and order of 240 together with the centrifugal field (m^2/s^2).

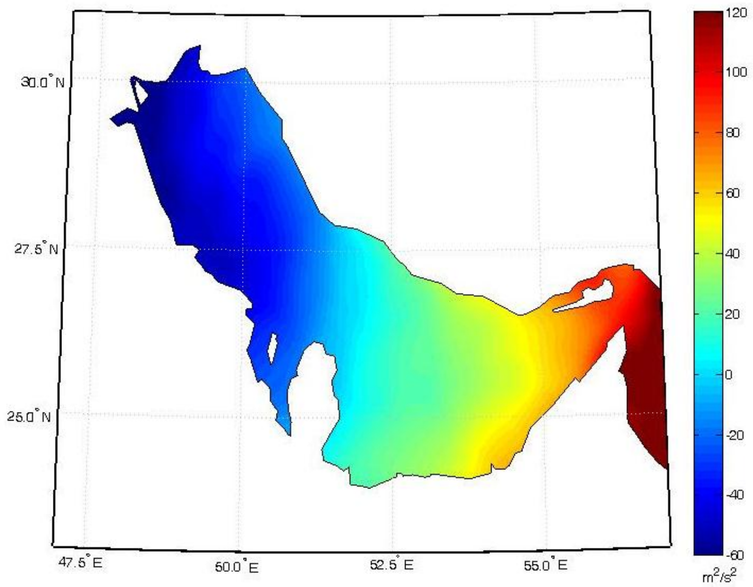

Figure 8. The residual potential over the Persian Gulf (m^2/s^2).

Residual potential satisfies the Laplace equation in the outer space of the boundary. The boundary $\sum$ of the problem is a regular surface of the reference ellipsoid. In the next step, we respond to the Dirichlet BVP using the residual potential as boundary data. To compute residual potential within and outside the boundary $\sum$, harmonic spline interpolation is applied. Determination of the optimal regularization parameter and the optimal radius of the Bjerhammer sphere is very important kernel expansion of the harmonic splines modeling. As has been pointed out in the previous section, α is the radius of the Bjerhammer sphere for which the optimal value is determined using the signal-to-noise ratio in modeling the boundary data at the reference ellipsoid surface. This ratio is used to choose the optimal filter; i.e., among different filters for removing the noise of the function, the filter with the highest signal-to-noise ratio (SNR) is selected. The SNR is given by the following relation:

$$SNR = 10\log_{10}\left(\frac{\sum_{i=1}^{N} S_i^2}{\sum_{i=1}^{N}\left(S_i - \hat{S}_i\right)^2}\right) \tag{4}$$

where S is the original function and $\hat{S}$ is the estimated one. For each given α, SNR must be computed. The location where the SNR is maximized for different values of α is the location of the radius of the optimal Bjerhammer sphere. Figure 9 shows variations of the signal-to-noise versus α parameter. Based on this criteria, optimum value for α parameter was selected to be 6315564.59 m.

In order to determine the optimal smoothing parameter for solving the system of observation equations (Eq.3), L-curve method is used (Hansen, 1998). Figure 10 displays variations of the regularization parameter in the L-curve method for the optimal parameter determination. The optimal value has been set to 8.8314×10^{-12}.

Residual potential $U_N^F(x)$ is estimated in the external space of the reference ellipsoid from the solution of Dirichlet BVP using the harmonic splines. We work in the framework of the Runge-Krarup, i.e. $U_N^F(x)$ is considered as a member of the function space of the regular harmonic functions outside the Bjerhammer sphere with radius α, which is completely located inside the topographic masses. It is taken as an approximation of the true residual potential at points over and outside the surface of the reference ellipsoid. Once the residual potential is estimated in the external space of the reference ellipsoid, it is possible to apply any linear operator to express other residual gravitational quantities (Jekeli, 2005). The residual gravitational acceleration is computed by application of the gradient operator to residual gravitational potential of former step. Figure 11 shows variations of the modulus of the residual gravity acceleration at the surface of the reference ellipsoid.

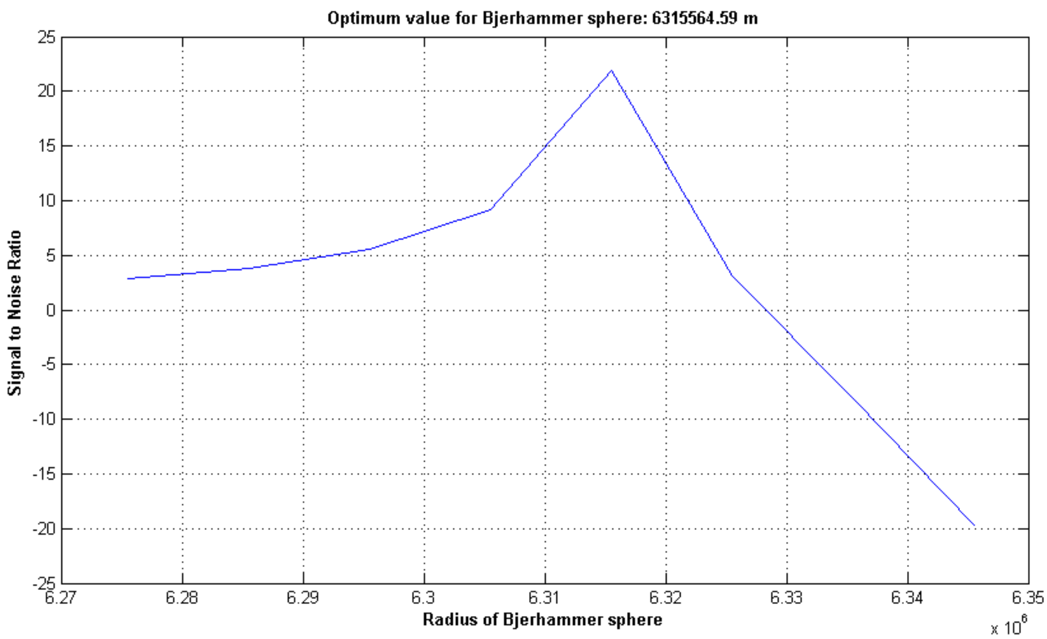

Figure 9. Variations of the radius of the Bjerhammer sphere with the signal-to-noise ratio.

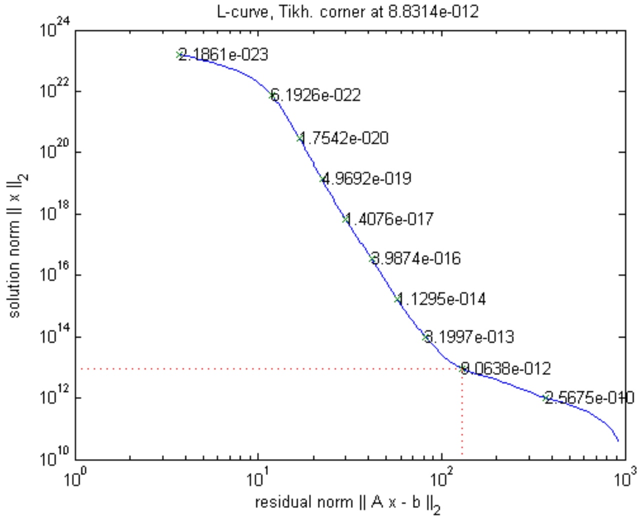

Figure 10. L-curve and the optimal regularization parameter.

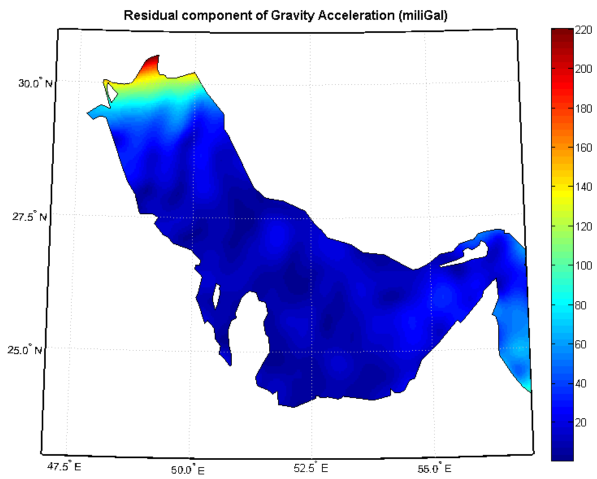

Figure 11. Residual gravity acceleration over the Persian Gulf (miliGal).

In order to obtain the gravity acceleration, we have to restore the effect of the reference gravity field. Figure 12 displays variations of the computed gravity acceleration in the test area over the surface of the reference ellipsoid.

Eventually, we can use these values to produce gravity acceleration data at he sea surface in the test area of the Persian Gulf. At our test area, the sea surface is under the reference ellipsoid (Figure 4), i.e., the boundary of the Dirichlet problem; consequently results are downward continued to the sea surface using free-air reduction. Figure 13 shows variations of the actual gravity acceleration at the sea surface of the test area.

Finally, the computed gravity acceleration has been tested for the shipborne gravimetry data. Low and sparse coverage of shipborne gravity data in the Persian Gulf is shown in Figure 14. The data of this region was provided by International Gravimetric Bureau organization. Table 2 summarizes statistics of the difference between the computed gravity acceleration and the shipborne gravity acceleration observations at 5311 stations in the test area.

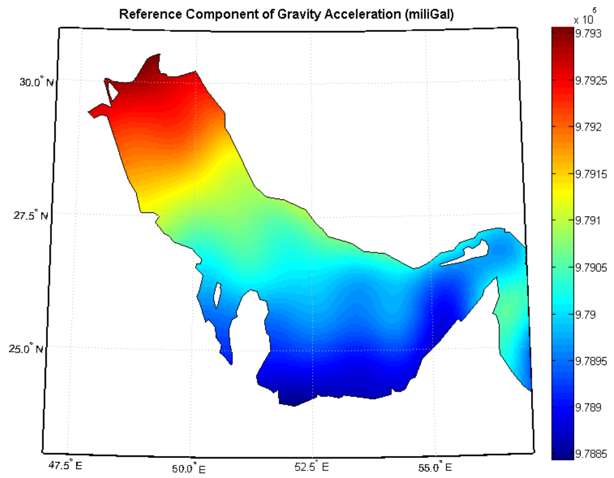

Figure 12. Reference component of the gravity acceleration over the Persian Gulf (miliGal).

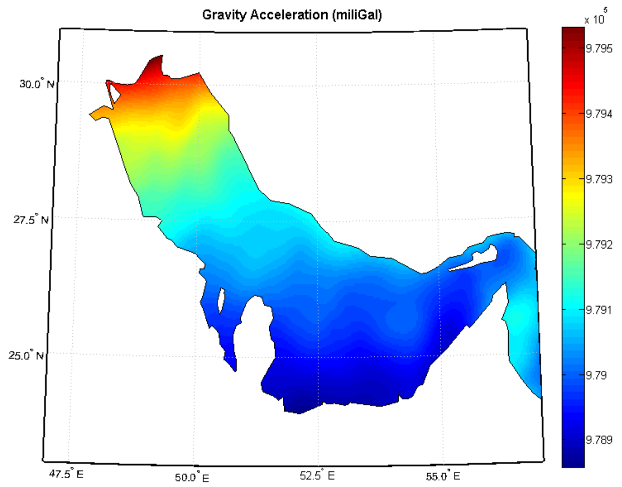

Figure 13. Gravity acceleration at the sea surface over the Persian Gulf (miliGal).

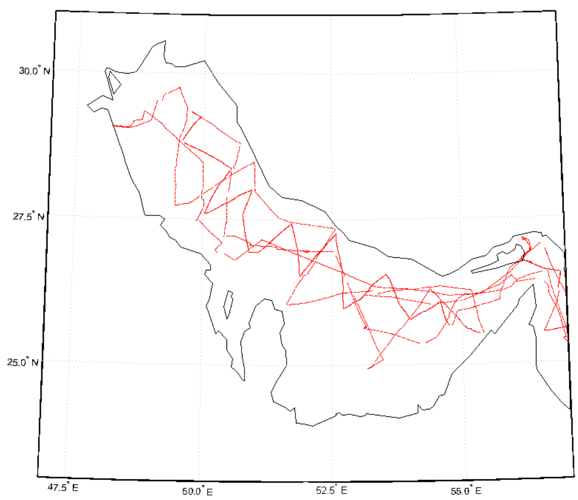

Figure 14. Trajectories of shipborne Gravimetry surveys over the Persian Gulf.

Table 2. Statistics of the difference between the computed gravity acceleration and 5311 shipborne gravity acceleration observations in the test area (in miliGal).

Maximum	**5.99**
Minimum	**-3.49**
Mean	**1.37**
STD	**2.61**

4 Conclusion

A quite general and still simple technique for production of the gravity acceleration at the sea areas based on satellite altimetry data and harmonic splines has been applied in this paper. According to the results obtained for the gravity acceleration over the Persian Gulf and differences between these obtained data and the shipborne gravimetry data (as it is shown in Table 2), it is concluded that the application of the satellite altimetry observations and the harmonic spline approach is a viable alternate for acquiring the marine gravity data.

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