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Simulation of entrainment near a density stratified layer: Laboratory experiment and LIDAR observations

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Abstract

In this paper a simple qualitative model of growth of a mixed layer adjacent to a uniform layer with a stably stratified layer is presented. The entrainment of a stably stratified layer into a turbulent mixed layer in a confined region was studied in laboratory for different Richardson numbers. The results for entrainment near a density stratified layer show that the entrainment rate is a weaker function of *Ri* than the case with a step density profile. It is also shown that the internal waves in the stratified region, with typical buoyancy period of about 10 *s* may interact with turbulence near the interface and create a non-uniform entrainment rate as an oscillatory behavior with a typical time scale of 150 *s*. The process is qualitatively consistent with processes associated with internal waves interaction with turbulence and create a momentarily buoyancy flux with different signs. Results of lidar observations of atmospheric boundary layer also show that the oscillation of aerosol layer is probably due to non-uniform entertainment in the interface between the mixed layer and free atmosphere. Although there seems to be a qualitative similarity between entrainment behavior at the top of the atmospheric mixed layer and laboratory experiments "mixing box", but the two are quite different as lab experiments are in an enclosure without mean flow, while the top of the mixed layer is a free solid boundary flow which may be associated with mean flow shear.

Keywords: Atmospheric mixed layer depth, Entrainment zone, Laboratory experiments

1. Introduction

In natural flows, turbulence can be generated by a variety of mechanisms, for example mean velocity shear, breaking of surface (in oceans) or internal waves, and thermal convection due to either heating of the ground (in the atmospheric flows) or cooling of the ocean surface (Turner, 1973). Specifically, it has been recognized that shear is a major source of mixing in natural flows (Fernando, 1988), in that it not only produces turbulence via interaction with Reynolds stresses but also can directly cause mixing at stratified interfaces by exciting Kelvin-Helmholtz (K-H) instabilities (Strang and Fernando, 2001).

The atmospheric boundary layer has a well-defined structure that evolves with the diurnal cycle (Stull, 1988). As shown in fig.1, in a diurnal cycle, after sunrise mixed layer (ML) grows and in the afternoon it subsides as often seen in remote sensing observations. The mixed layer is usually capped by an entrainment zone (EZ), which provides a transition to the often stable lower troposphere (Garrat, 1992; Gryning and Batchvarova, 1994). The study of the entrainment in atmospheric boundary layer has many applications, from the parameterizations of entrainment used by forecast models to its impact on large scale dynamics. Entrainment processes are complicated and are often associated with shear, convection, internal waves breaking etc (Thorpe, 1973; Lin and Linden, 2005). However, it is difficult to measure the entrainment rate directly (Gryning and Batchvarova, 1994; Deardorff et al., 1980; Bretherton et al., 1999; Flamant et al., 1997; Mok and Rudowicz, 2001; De Rooy and Siebesma, 2010; De Rooy et al., 2013; Jonker and Jimenez, 2014).

The entrainment of a stably stratified layer into a turbulent mixed layer in a confined region has been studied in laboratory for different Richardson

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numbers (Kato and Philips, 1969; Cardoso and woods, 1993). The internal waves generated at the interface are transmitted into the stratified fluid (Deardorff et al., 1980). The modal structure of these waves appears to interact with the turbulence processes near the interface creating a non-uniform entrainment rate usually in steps. This may be related to the vertical wave number of the dominant wave which is dependent on the depth of the stratified layer as well as the horizontal cross section of the tank. A similar behavior is also observed in the deepening of plume outflow entering a preexisting stratified fluid in a confined region (Deardorff et al., 1980).

Density interface (DI) between mixed regions and stratified fluids is ubiquitous in geophysical fluids as in atmospheric mixed layer during day or oceanic mixed layer. The processes of entrainment near interfaces with step density profiles have been studied extensively (e.g. Linden, 1980; Turner, 1973; McGarth et al., 1997; Strang and Fernando, 2001; Da Silva et al., 2014), however such processes near a DI which is adjoined a density stratified region, similar to real atmosphere and ocean has less been studied.

The processes of turbulent entrainment near a sharp DI may include eddy impinging, engulfment, Kelvin-Helmholtz (K-H) instability and breaking of internal waves at the interface (McGarth et al., 1997). However in the case with a density stratified region the modal structure of internal waves propagating into the stratified zone that depends on the depth of this region, may also have an influence on the entrainment. The main none dimensional parameter controlling the entrainment velocity u_e, is the bulk Richardson number ($Ri_b = g\Delta\rho l/(\rho_o u^2)$) where g is the gravitational acceleration, $\Delta\rho$ is the density step near the DI, which can be $\sim l\partial\rho/\partial z$, where l is a turbulence length scale, $\partial\rho/\partial z$ is the density gradient in the stratified region, ρ_o is the mean density of the environment and u is a turbulence velocity scale that may be changing as a result of interaction of internal waves and turbulence near the interface (Strang and Fernando, 2001). Interaction of internal waves with turbulence near the DI appear to generate time varying buoyancy flux which in certain cases may become negative (counter gradient processes) as have been observed in grid generated turbulence in a stratified environment (e. g. Itsweire et al., 1986; Bidokhti and Britter, 2002).

The objective of this research is to simulate the growth of a mixed layer near a density stratified layer in a confined region using laboratory experiments. The observed processes are also compared with some entrainment process measured in the entrainment of an atmospheric boundary layer.

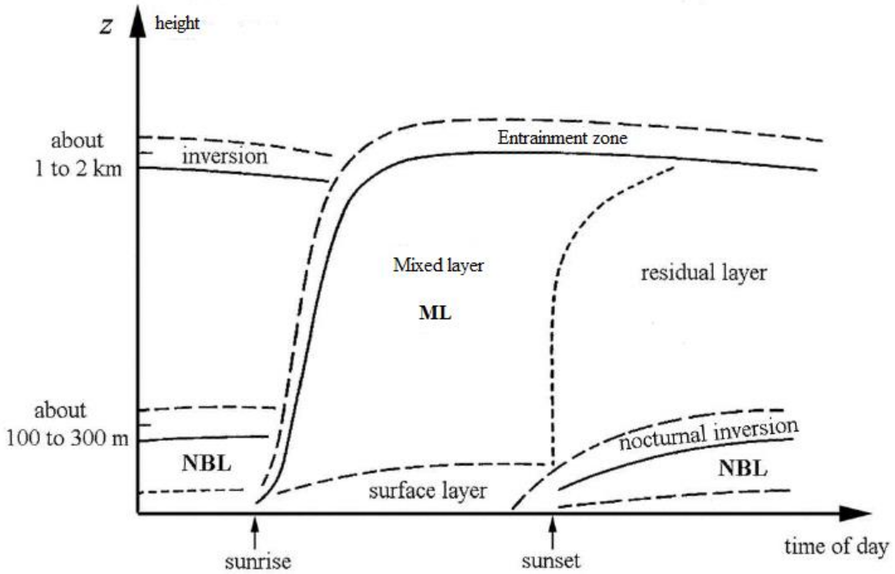


Figure1. Evolution of atmospheric boundary layer (ABL) over land and under clear skies. At sunrise, heating from below sets to a mixed layer (ML), while at sunset convection is terminated and a nocturnal boundary layer (NBL) is formed. (Garratt, 1992).

2. Methods

The experiments are performed in a rectangular water tank with the size of $40\times 40\times 70cm^3$, as sketched in Fig. 2. The satisfaction is produced by the double bucket method; the stratified layer of fluid is separated by a thin interface. The lower part of the tank is stratified using salt solution of depth 28 *cm* and a layer of homogeneous fluid with the same depth was gently introduced over the stratified lower layer. The vibrating grid in the top homogeneous layer is initially at 15 *cm* from the interface between the layers. The grid is a monoplanar horizontal one with grid size of 5 *cm* that oscillates with a small stroke of 1.5 *cm* and a frequency of 2 *Hz*. Hence the turbulence velocity and length scale are given respectively by $u=\alpha\omega/z$ and $l=\beta z$ where α and β are coefficients and in CGS (metric system) for this condition are 0.56 cm^2 and 0.1 respectively (Linden, 1980). ω is the oscillation frequency of the grid and z is the mean distance of the grid from the interface. Typical density profile of fluid before the start of the experiment is shown in Fig. 3a.

The Richardson numbers used in the experiments were sufficiently large ($Ri>1$), favorable for the usual eddy-engulfment mechanism. Five experiments were carried out in which the flow geometry and grid oscillation were the same, but the stratification of the stratified layer was systematically changed. The buoyancy frequencies of the stratified layer in these experiments were 0.77, 1.18, 1.03, 1.27 and 1.51 s^{-1}. The entrainment speed was measured by direct observation of the interface movement with time i.e. the erosion of the stratified layer by entrainment processes and encroachment. The mixed layer was dyed as to make the interface distinct. The dye crystals dropped into the tank creates vertical lines that reveal the modal structure of the waves as sinusoidal vertical curves in the stratified region.

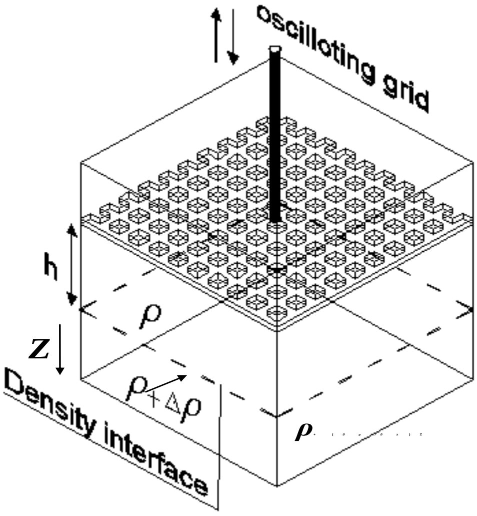
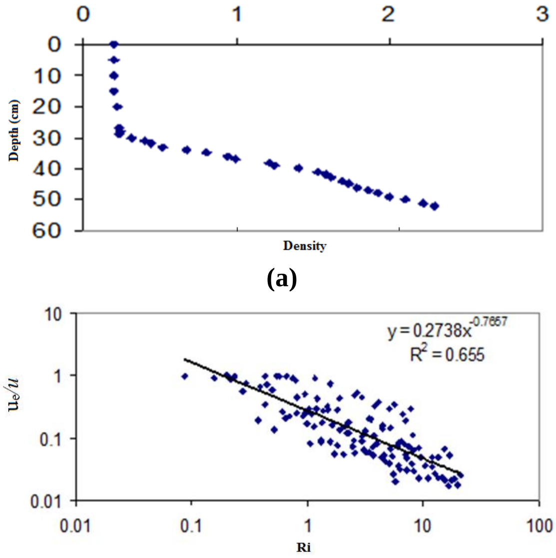


Figure 2. Experimental set-up i.e. a quibic water tank with the size of $40\times 40\times 70cm^3$.



(a)

(b)

Figure 3. (a) Typical vertical profile of density, (b) variation of normalized entrainment speed with R_i for all experiments.

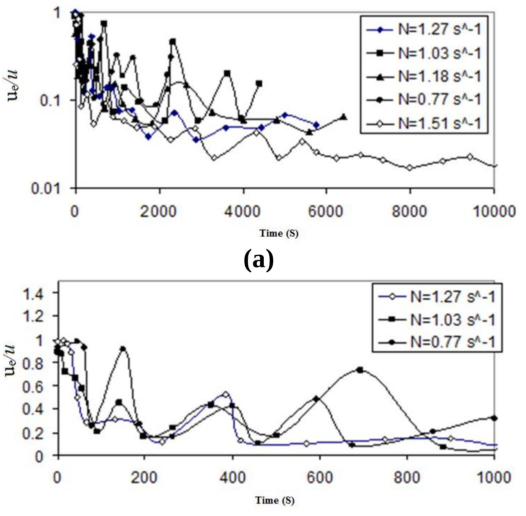
To examine the applicability of these experiments for the atmospheric boundary layer, a 532-nm elastic backscatter LIDAR located at the Institute for Advanced Studies in Basic Sciences of Zanjan (48.5°N, 36.7°E) was used. Aerosol backscattering from the convective boundary layer shows spatial variations due to non-uniform mixing of the naturally occurring aerosol layer near the entrainment zone. A sample day (e.g. 12 May 2006) has been chosen based on the availability of data.

3. Results and discussion

The variations of the entrainment speed at the interface normalized by a turbulence velocity near the interface (i.e. u_e/u), versus Ri for all experiments are shown in Fig. 3b. A power law of $u_e/u=c\ Ri^{-0.76}$ can be fit to the data. This shows that there is a weaker dependence onf Ri than for the case with a step density profile (with Ri^{-1}) between two uniform density layers with a density step. This may indicate that part of the turbulent energy goes to internal waves in the stratified region and propagate into the fluid bulk and boundary of the tank instead of going in to be mixed at the interface. This phenomenon is also likely due to wave breaking, which takes over interfacial mixing and turbulent energy transfers to stratified region. It should be stressed that up to seventy percent of the turbulent kinetic energy is also dissipated by viscosity (e.g. Itsweire et al., 1986).

The more interesting aspect of the result is the inter-variability i.e. non-uniformity of the entrainment at the interface. Fig. 4 shows u_e/u versus time in all experiments. It emphasises the non-uniformity in the entrainment speed with time. The scatter of data is likely due to variability in the rate of entrainment which may be coherent in the experiment. There are oscillations in the entrainment rate with time which are not regular. It is likely due to measurement error that are typically about 20 percent.

Fig. 5 shows the non-dimensional entrainment speed u_e/u versus the non-dimensionalized time Nt (by the buoyancy frequency of the stratified region $N= [-g/\rho_o(\partial\rho/\partial z)]^{1/2}$ is typically about 1.1 s^{-1}) in some such experiments, showing the non-uniformity in the entrainment speed with time. There are oscillations in the entrainment speed with time which is coherent in phase. The non-uniform entrainment at the interface may be related to the interaction of the internal waves (their modal structures) in the stratified layer (Gill, 1982) and turbulence at the interface. The buoyancy period in these experiments is in the range of 5-15 s. The turbulence time scale (~l/u) at a distance of about 20 to 30 cm from grid, have the same value. Interaction of internal waves with turbulence near the DI in these experiments appear to generate time varying buoyancy flux which in certain moments may become negative (counter gradient processes) as have been observed in grid generated turbulence in a stratified environment (e. g. Itsweire et al., 1986; Bidokhti and Britter 2002). Regular variation of entrainment rate at the interface should indicate that the buoyancy flux should also vary. Stronger entertainment should coincide with larger buoyancy flux. Locally mixed fluid generated by interfacial instabilities causes the interface to swell, leading to an intermediate layer sandwiched between the interfacial layer and the upper mixed layer, which, in turn, is eroded by the mixed-layer eddies to generate a low-frequency oscillation (McGarth et al., 1997).



(a)

(b)

Figure 4. (a) variation of entrainment speed against time for all experiments; (b): same as (a) but for only three experiments and their early stages.

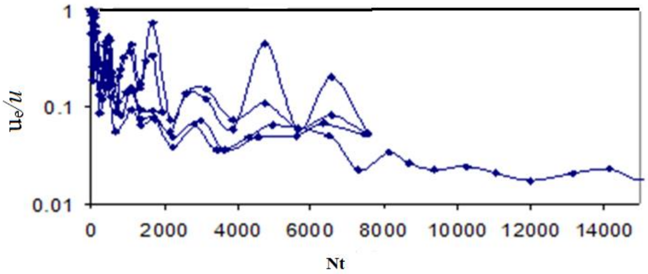


Figure 5. The variation of entrainment speed against none dimensional time for all experiments.

The ensuing thin interface again becomes unstable, causing K-H billowing and swelling. The recurrence of these events acts as the excitation source of internal waves. When *Ri* is increased, the production rate of intermediate-density fluid is faster than or of the same order as its removal by the eddies and thus the intermediate layer is well defined. Estimation of internal-wave radiation based on the frequency of the swelling phenomenon is consistent with that evaluated by the direct measurement of the energy budget.

The mutually resonating K-H and asymmetric waves, appears to be responsible for the higher entrainment rates (interfacial erosion), observed for the two-layer case and largest disparity of entrainment rates observed between two-layer and linearly stratified cases. If we also tentatively assume that in these experiments we have an establishing time that is equal to this value times the buoyancy period, it would be about 150-200 *s* which appears to be similar to the oscillation period, at least at the early stages of the experiments (Fig.4b). Establishing time is the time that the internal wave field adjusts to new condition that is quasi-steady, for times that waves travel the length of the tank (Wong et al., 2001).

To examine the applicability of these experiments to the observation of atmospheric boundary layer behavior, we use an elastic backscatter LIDAR. In the early morning when the atmospheric mixed layer depth (AMLD) is shallow, small air volume has high pollutant concentrations in the early morning. As AMLD increases towards midday the concentration is substantially reduced (Hanna, 1982). Entrainment zone (EZ) is occasionally accompanied with large entrainment mixing due to strong vertical shear. Aerosol backscattering from the convective boundary layer shows spatial variations due to non-uniform mixing of the naturally occurring aerosol. In some instant two different aerosol layers-Mixed layer and Residual layer could be obviously seen in Fig. 6. During this experiment with high entrainment, as shown in figure (7a), the mixed layer growth started at about 12:00 to 13:00 LT (local time) with a growth rate of around 200 to 300 mh^{-1}. As a result, the internal waves with the non-uniform entrainment of mixed layer and free atmosphere with the amplitude of around 100 meter and a period of 15 minutes, is formed. Subsidence of the mixed layer could be seen in figure (7b). It shows that a turbulence reduction with a rate of 100 mh^{-1}, starting from 16:30 LT. Considering figure (7b), the amplitude and the period of K-H waves were measured around 50 meters and about 20 minutes at around 18:00 LT. It is notable that the entrainment process had been started from the sunrise (about 7:00 of LT) with a 100-meter wave amplitude and 15-minute period, while it had been continued to the end of the day (Figure 7c). The non-uniform entrainment at the interface may be related to the interaction of the internal waves (their modal structures) in the stratified layer and turbulence at the interface. It is notable that internal waves which are generated in interface are transmitted the turbulence energy that may go to the stratified region (free atmosphere).

Entrainment is a result of turbulence driven by the surface heat flux and turbulence generated by wind shear (Stull, 1988). Thermals, which are positively buoyant at the surface, rise through the mixed layer until they reach the warmer free atmosphere and become negatively buoyant; they overshoot a small distance because of strong static stability in EZ and fall back towards the mixed layer. Strong wind shear in the EZ can lead to instability and turbulence if Richardson number become smaller than 1/4. Interaction between internal waves with turbulence near the DI causes non-uniform entrainment in the entrainment zone. These non-uniform entrainments generate periodic wave's growth and breaking waves at the interface of mixed layer and the free atmosphere. Intensive vertical wind shear at 6:00 am causes the internal wave generation. Following that, the breaking of the waves happens due to the interaction of the mixed layer turbulence with the internal waves. This process is the key parameter to growth of the mixed layer depth more rapidly rather than that for the next hours. The higher depth of mixed layer in this hour may be reasonable (Brooks, 2003). The waves would be dissipated by weak wind shear in the next hours. As free atmosphere density is usually stratified, non-uniform entrainment process in interface between free atmosphere and mixed layer is accompanied by breaking internal wave in entrainment zone. Generated turbulence due to mixed layer plumes, convection and interaction with internal waves at interface may cause a fraction of turbulence energy, being transmitted to free atmosphere.

Oscillation of aerosols layer is probably due to non-uniform entrainment in interface between the mixed layer and the free atmosphere exhibited by increasing range resolution of LIDAR in entrainment zone. It was shown that results in experiments had some consistency with growth and aerosol layer oscillations in atmospheric boundary layer. Although there seems to be a qualitative similarity between entrainment behavior at the top of atmospheric mixed layer and that of laboratory experiments but the two are quite different as lab experiments are in an enclosure without mean flow, while top of the mixed layer is a free solid boundary flow which may be associated with mean flow shear.

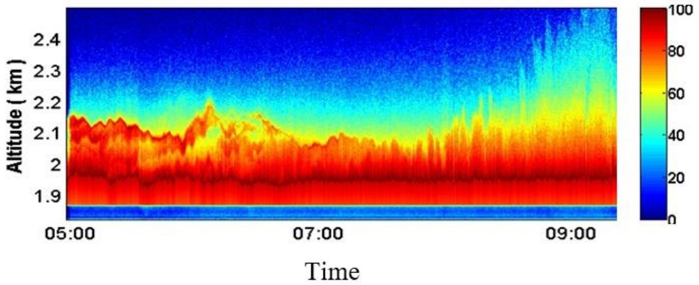
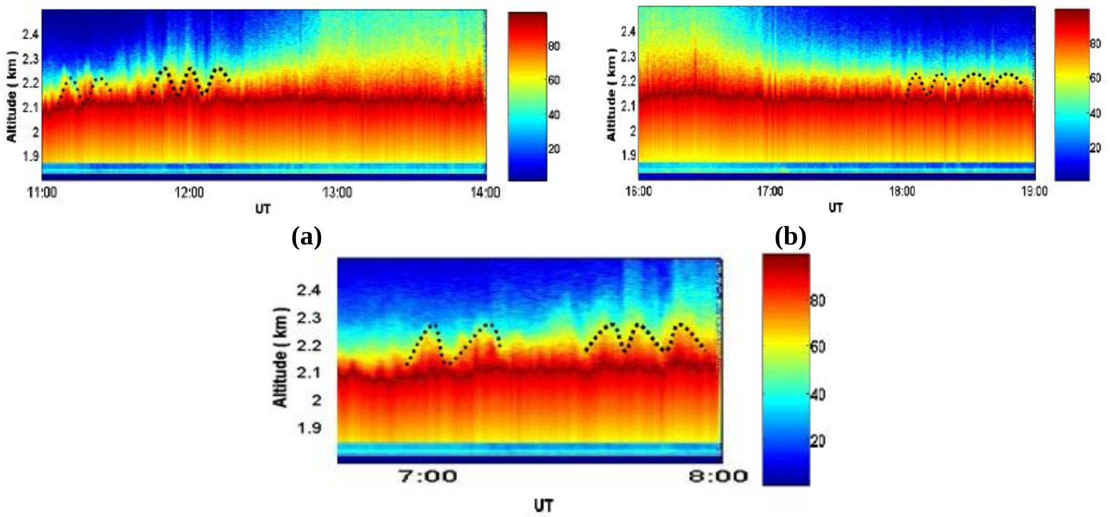


Figure 6. Boundary layer growth from 0500 to 1100 UTC of 12th of May 2006. LIDAR(532 nm) (Note that the dead zone in this fig. is about 100m).



(a) (b)

(c)

Figure 7. growth rate of around 200 to 300 mh-1(a). Subsidence of the mixed layer, turbulence reduction with a rate of 100 m/h-(b), from 16:30 LT. (b), Entrainment process has started from the sunrise to the end of the day (c).

4. Conclusions

The entrainment of a stably stratified layer into a turbulent mixed layer in a confined region was studied in laboratory for different Richardson numbers. The experiments are performed in a rectangular water tank. The lower part of the fluid in the tank is stratified and a homogeneous layer of fluid was gently introduced over the stratified layer. The entrainment processes were studied in this so called mixing box and turbulence was generated via vibrating grid near the density interface.

The experiment results showed that the entrainment rate was a weaker function of *Ri* than that for the case with a step density profile between two uniform density layers with density step. It was also shown that the internal waves in the stratified region, with typical buoyancy period of about 10 *s* might interact with turbulence near the interface and create a non-uniform entrainment rate as an oscillatory behavior with a typical time scale of 150 *s*. The process is qualitatively consistent with processes that internal waves interact with turbulence and create a momentarily buoyancy flux with different signs (Pham and Sarkar, 2010). The modal structure of these waves appears to interact with the turbulence processes near the interface creating a non-uniform entrainment rate usually in steps. This may be related to the vertical wave number of the dominant wave which is dependent on the depth of the stratified layer as well as the horizontal cross section of the tank. The results show that the dominant mixing mechanisms are different for different Richardson number ranges, the dominant entrainment mechanism is the impingement of eddies on the interface and splashing of heavier fluid into the mixed layer.

To examine the applicability of these results to the atmospheric boundary layer, a 532-nm elastic backscatter LIDAR was used. Aerosol backscattering from the convective boundary layer shows spatial variations due to non-uniform mixing of the naturally occurring aerosol layers near the entrainment zone. The internal waves with the non-uniform entrainment of the mixed layer and free atmosphere with the amplitude of around 100 meters and a period of 15 minutes, is formed. The amplitude and the period of K-H waves are measured around 50 meters and about 20 minutes respectively at around 18:00 LT. The entrainment process was started from the sunrise with a 100-meter amplitude and 15-minute period, while it continued to the end of the day. The non-uniform entrainment at the interface may be related to the interaction of the internal waves in the stratified layer and turbulence at the interface. It is notable that internal waves which are generated in the interface transmit the turbulence energy in to the stratified region (free atmosphere).

Results show that the oscillation of aerosol layer is probably due to non-uniform entertainment in the interface between the mixed layer and free atmosphere with an increasing range resolution of LIDAR in entrainment zone. The experimental result has some consistency with the growth and oscillations of aerosol layer in atmospheric boundary layer. Although there seems to be a qualitative similarity between entrainment behavior at the top of the atmospheric mixed layer and laboratory experiments but the two are quite different as lab experiments are in an enclosure without mean flow, while the top of the mixed layer is a free solid boundary flow which may be associated with mean flow shear.

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