

## Self-consistent hot spot tracing by kinetic simulations: with the emphasis on Cusp particle entry

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### Abstract

One of the most important advantages of particle simulation as compared to fluid simulation is the capacity for working with and tracing particles. In particle simulations, the test particle method is usually used to get some idea of the behavior of plasma or other substances. In this method, first, a small number of particles are injected into the frame of static electromagnetic fields. Then, movement of particles is investigated using the pattern of the electromagnetic fields. This method is useful; however, as it is needed to work with non self-consistent fields, it lacks precision. In this work, we adapted the particle simulations method, adding the flexibility of working with self-consistent fields that come directly from the simulation. Here we have tried to investigate particle entry from the solar wind with northward Interplanetary Magnetic Field (IMF) to the magnetospheric cusp. Here we tried to trace several particles to the high altitude cusp in a self-consistent manner. To benchmark the self-consistent tracing, we compared our results and the results of test particle method with the observational data from CLUSTER satellite.

**Keywords:** Self-consistent hot spot tracing, Particle In Cell (PIC) simulation, Cusp particle entry, Test particles method, IMF, Solar wind, Magnetosphere.

### 1. Introduction

The test particle method involves first injecting a limited amount of particles randomly into the static(dynamic) frame of the electromagnetic field resulting from the simulation, and then examining how these particles move and react in response to the background electromagnetic fields (Hur and Suk, 2007). In this method, as the number of injected particles is small, fields will not change due to the particles. This method gives a good idea of particle dynamics. However, for capturing the self-consistent physics of the simulation and achieving better insight into the problem, it is better to include the field's dynamics. In this work, our aim was to see particle dynamics with self-consistent electromagnetic fields. Tracing particles in normal PIC simulation has many potential applications. For example, tracing particles could improve investigation of particle transportation via magnetic reconnection in plasma astrophysics and observing patterns of particle dynamics in different field topologies. Tracing one or several particles in PIC codes is common, but to our

knowledge, there has been no particle tracing for hot spots in PIC codes. By hot spot, we mean the important area(s) in the simulation that we are interested to investigate the physics of that area. Here we had only a simple hot spot tracing from solar wind to the cusp. Our initial results with the maximum density (this could be energy, temperature and other parameters as well) in the cusp as a hot spot, shows the path through which particles mostly enter the cusp.

### 2. Simulation model

In our 3D simulation, which was done using self-consistent TRISATN (3D Stanford) code developed by Buneman, we used the same initial conditions to form the magnetosphere (Buneman 1993; Buneman et al. 1995). The Earth dipole field is located inside the simulation domain. The rough size of Earth magnetosphere which is small enough to be inside the simulation domain is calculated. Radiating boundary conditions (Lindman 1975) and charge-conserving formulas (Villasenor and Buneman 1992;

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Nishikawa 1997) as in the works of Nishikawa (Nishikawa1998; Nishikawa and Ohtani 2002) are used. By radiating boundary conditions we mean, for each created electron at the boundary and injected inwards, the code assumes that a positive charge has been created at the same spot. The grid size is  $\Delta \cong 0.2RE$ , and  $\Delta t \cong 1$  is the time step ( $\omega_{pe}\Delta t \cong 0.12$ ). Here  $\Delta = \Delta x = \Delta y = \Delta z$ .

Initially, we use about  $2.2 \times 10^8$  electron-ion pairs, which corresponds to a uniform particle density of  $\tilde{n} = 8.0$  pairs per cell across the simulation domain ( $365\Delta \times 275\Delta \times 275\Delta$ ). Here “ $\sim$ ” denotes the normalized parameters defined as (“e, and i” denotes electron and ion, respectively):

$$\text{Thermal velocity } \tilde{v}_{the,i} = \frac{v_{the,i}}{\Delta / \Delta t} \quad \text{Debye length: } \tilde{\lambda}_{De,i} = \frac{\tilde{v}_{the,i}}{\tilde{\omega}_{pe,i}}$$

$$\text{Larmor gyroradius: } \tilde{\rho}_{ce,i} = \frac{\tilde{v}_{the,i}}{\tilde{\omega}_{pe,i}}, \quad \text{Inertia length:}$$

$$\tilde{\lambda}_{ce,i} = \frac{\tilde{c}}{\tilde{\omega}_{pe,i}}, \quad \text{Gyrofrequency: } \tilde{\omega}_{ce,i} = \frac{\omega_{ce,i}}{(\Delta t)^{-1}} = \frac{\tilde{B}\Delta m_e}{\Delta t m_{e,i}}$$

$$\text{Plasma frequency: } \tilde{\omega}_{pe,i} = \frac{\omega_{pe,i}}{(\Delta t)^{-1}} = \sqrt{\frac{\tilde{q}_{e,i}^2 \tilde{n}_{e,i}}{\epsilon_0 m_{e,i}}} \Delta t,$$

$$\text{Gyroperiod: } \tilde{\tau}_{ce,i} = \frac{2\pi}{\tilde{\omega}_{ce,i}}, \quad \beta \text{ ratio: } \tilde{\beta}_{e,i} = \frac{\tilde{T}_{e,i} \tilde{\omega}_{pe,i}^2}{\tilde{B}^2}$$

values of scaled ambient plasma parameters used in our simulation are following:

$$\tilde{v}_{the,i} = (0.09, 0.045), \quad \tilde{\lambda}_{De,i} = (1.4, 2.8), \quad \tilde{\omega}_{pe,i} = (0.125, 0.031),$$

$$\tilde{\omega}_{ce,i} = (0.08, 0.005), \quad \tilde{\rho}_{ce,i} = (1.56, 12.6), \quad \tilde{\lambda}_{ce,i} = (4.2, 16.1),$$

$$\tilde{\tau}_{ce,i} = (78.5, 1256), \quad \tilde{\beta}_{e,i} = (0.2, 0.8), \quad \tilde{T}_{e,i} = (0.008, 0.032).$$

The center of the current loop that generates the dipolar terrestrial magnetic field is located at  $(160\Delta, 137\Delta, 137\Delta)$ . Within the time range  $0 < \tilde{t} < 1000\Delta\tilde{t}$ , a drift velocity  $\tilde{v}_{sol} = 0.5\tilde{c}$  representing the solar wind is applied along the x-direction without an IMF. Here  $\tilde{c} = 0.5$  is the speed of light and its value determined using Courant condition ( $\delta x, \delta y, \delta z > c\delta t$ ).

The injected solar wind density also has  $\tilde{n} = 8.0$  electron-ion pairs per cell, the mass ratio is  $m_i / m_e = 16$  (Cai et al., 2003, 2006). All the normalizations and parameters are summarized in Table 1.

**Table 1.** List of normalizations and parameters

Parameter	Normalization	Electron	Ion
Thermal velocity	$\tilde{v}_{the,i} = \frac{v_{the,i}}{\Delta / \Delta t}$	<b>0.125</b>	<b>0.0625</b>
Debye length	$\tilde{\lambda}_{De,i} = \tilde{v}_{the,i} \tilde{\omega}_{pe,i}^{-1}$	<b>1.4</b>	<b>2.8</b>
Larmor gyroradius	$\tilde{\rho}_{ce,i} = \frac{\tilde{v}_{the,i}}{\tilde{\omega}_{pe,i}}$	<b>1.56</b>	<b>12.6</b>
Gyrofrequency	$\tilde{\omega}_{ce} = \tilde{B}, \tilde{\omega}_{ci} = \tilde{B} r_{mass}$	<b>0.08</b>	<b>0.005</b>
Inertia length	$\tilde{\lambda}_{ce,i} = \frac{\tilde{c}}{\tilde{\omega}_{pe,i}}$	<b>4</b>	<b>16</b>
Plasma frequency	$\tilde{\omega}_{pe,i} = \frac{\omega_{pe,i}}{\Delta t^{-1}} = \sqrt{\frac{\tilde{q}_{e,i}^2 \tilde{n}_{e,i}}{\epsilon_0 m_{e,i}}} \Delta t$	<b>0.125</b>	<b>0.031</b>
Gyroperiod	$\tilde{\tau}_{ce,i} = \frac{2\pi}{\tilde{\omega}_{ce,i}}$	<b>78.5</b>	<b>1256</b>
Temperature	$\tilde{T}_e = 2\tilde{v}_{th,e}^2, \tilde{T}_i = \frac{2}{r_{mass}} \tilde{v}_{th,i}^2$	<b>0.008</b>	<b>0.032</b>
<b>Parameters</b>			
Grid size	$\Delta \cong 0.2Re (= \Delta X = \Delta Y = \Delta Z)$		
Time step	$\Delta t \cong 1 (\omega_{pe}\Delta t = 0.12)$		
Interplanetary Magnetic Field(IMF)	$B_{IMF} = 0.08$		
Particle density	$n_e = n_i = 8$		
Speed of light	$\tilde{c} = 0.5$		
Solar Wind Velocity	$v_{sol} = 0.2$		
Alfven Mach Number	$V_A = 2.6$		
Plasma Beta	$\beta = 12$		

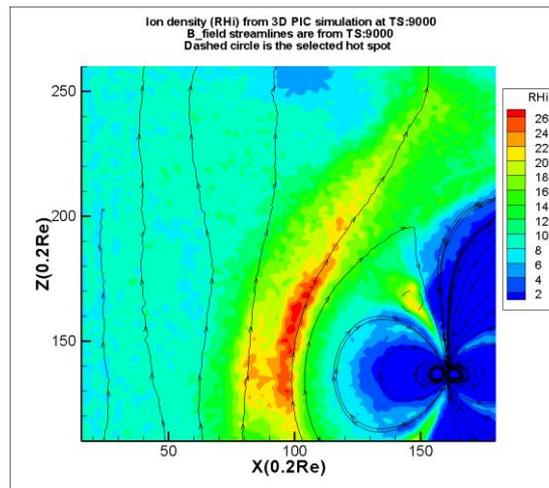
### 3. Method

In the hot spot tracing method, we first determine the hot spot which we mean the region of interest for example a region with maximum or minimum value in density or energy (which is obtained from PIC simulation), and then we select particles including ions or electrons in this hot spot. In the next step, we re-run the same code with the same amount of particles. All parts of the code initialization, including particle number and fields, remain the same; the only change is that this time the code will re-run with keeping traces of tagged particles. Then we see how particles move to the area of interest.

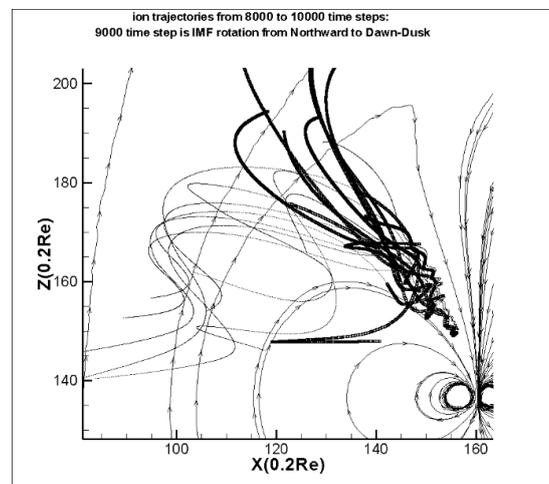
In this method, first we determine the hot spot in the simulation outputs as can be seen from the following figure (Fig.1).

In the red dashed circle area (Fig. 1), we have the maximum high altitude cusp density. Researchers are interested to see how this hot spot forms during solar wind interaction with the earth's magnetic field and to observe how particles are transferred via the reconnection region to the cusp.

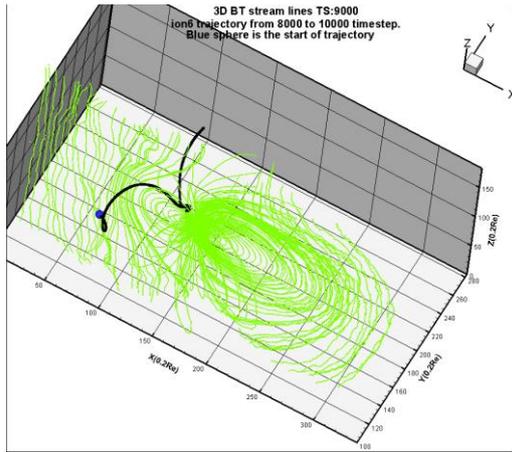
Secondly, we mark the particles (ions or electrons) in this area, and then go back a number of time steps in the simulation (for example 1000 time steps). Next tracing these marked particles, we see how they are pushed by the Lorentz force in the system. We can directly see how this density hot spot is formed. By tracing about 10 particles in the dashed circle from 1000 time steps before, we have the following results (Fig.2), and the 2D view of a particle out of 10 particles is shown in Fig 3:



**Figure 1.** Ion density, global 3D PIC simulation with northward IMF direction at the time step 9000. Here the hot spot is the marked circle, which is the maximum density in the high altitude cusp.



**Figure 2.** Results of particle tracing in 3D PIC simulation from 8000 to 10000. The points are the paths from 8000 to 10000 time steps and highlighted squares are the trajectories from 9000 to 10000 time steps. As the plot shows, particles are gathered at time step: 9000.



**Figure 3.** Results of particle tracing in 3D PIC simulation for a sample particle in figure 2, 3D view

Above (Fig. 2) are the self-consistent ion trajectories in the X-Z plane. For clarity, results for X-Y and Y-Z plots are not shown. 3D trajectory of a sample particle is shown in Fig. 3. Green streamlines in fig. 3 are the magnetic field streamlines which includes magnetosphere and IMF.

We investigated the results in comparison to the Interplanetary Magnetic Field (IMF) rotation. In this simulation, IMF is built in 3000 time steps with the northward direction and rotated smoothly to Dawn-Dusk from 9000 to 9300 time steps (for 300 time steps) (Cai et al. 2006). These results are in a good agreement with experimental observations that show the magnetospheric cusp dynamics relation to the IMF rotations (Lavraud et al. 2005). This tracing shows the relation of IMF rotation and particle entry to the cusp. Particle entry also can be investigated using plasma currents or the test particle method. However, using the tracing method gives much more direct and more exact vision as self-consistent fields (using original simulation) with the same number of particles have been used.

The concise algorithm for the hot spot tracing method is as follows:

- 1- Selecting a hot spot in the simulation.
- 2- Selecting particles (ions or electrons) by their position in the considered area.
- 3- Tagging particles in the simulation.
- 4- Running the simulation from enough time steps before the intended time and tracing the tagged particles.

#### 4. Comparing with test particle method results

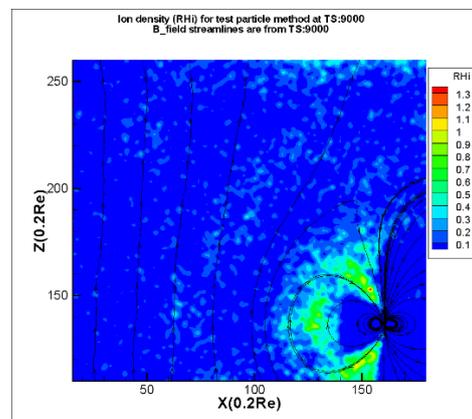
We compared the above results with the results of the test particle method. About 3 million pairs of ion-electrons used for the test particle method. Particles were injected evenly into the box at 8000 time step; simulation continued until 9000 time step. Figure 5 shows the results for non-self-consistent test particle method:

As can be seen from Fig.4 the test particle method also shows the hot spot in the cusp region. However, the spot is located at lower altitude as compared with the original PIC simulation in Fig.1. Furthermore, when the static EM fields (test particle method) is used, it is impossible to catch the physics relating to IMF rotation, which is important in solar-terrestrial simulation, but it is possible in case of tracing method.

In the case of the proposed tracing particle method, we need only to trace particles about 1000 time steps back, which is the size of simulation space. As shown in Figure 2, tracing from 8000 time step would be enough and gives the required physics information.

Test particle method is good in terms of computational equipment and timing as it requires tracing of few particles ( $3 \times 10^6$  ion-electron pairs) compared to 1 run of PIC simulation ( $2.2 \times 10^8$  ion-electron pairs, but tracing a few is required), but as it is not possible to work with self-consistent fields, the accuracy decreases a little.

Hot spot tracing may be expensive in terms of timing, but it has good accuracy as it has the capability to work with self-consistent fields of the PIC code.

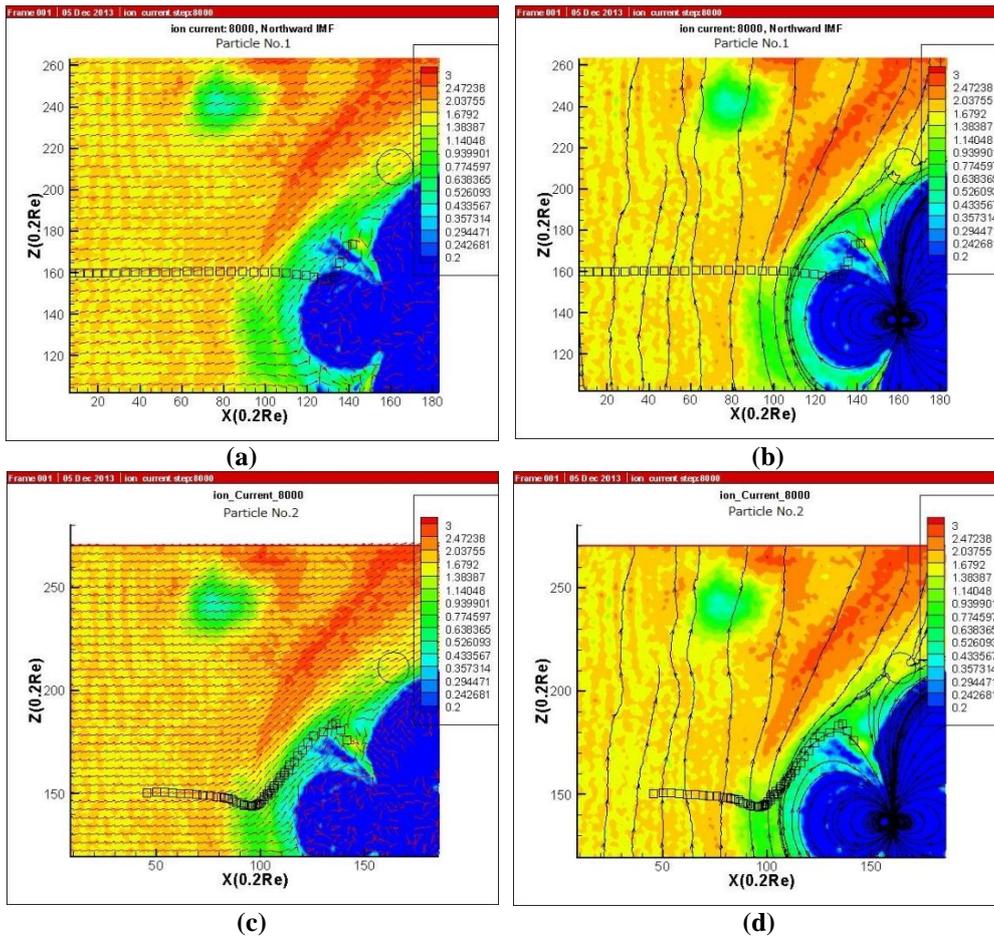


**Figure 4.** Results of Test particle method with the initial EM fields of 9000 time step and about  $3 \times 10^6$  electron-ion pairs

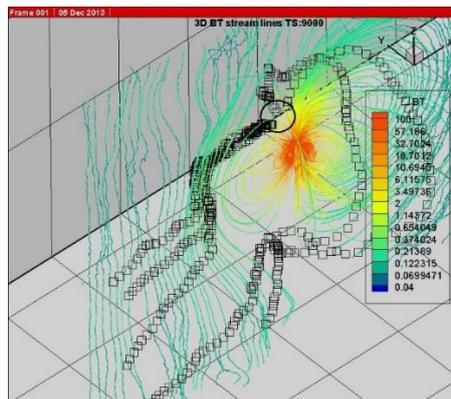
**5. Comparing with currents method for investigation of cusp particle entry**

Currents from PIC or MHD simulation are good tools for investigation of particle entry. Here we compared the trajectory of 2 particles with the path from currents in Fig.5. The IMF is northward and the magnetic field streamlines are plotted by black lines. In this self-consistent method, particle trajectory (squares in Fig. 5b and d) from the simulation can give an exact path

from the solar wind to the cusp, but as can be seen from the background ion currents (arrows in the figures 5a and c) near to the cusp, it is too noisy and we cannot have the exact image of particle entry to the cusp. The circle in the figures is the place of x-point in the tailward-side reconnection region during northward IMF. For particle 2 in Fig 5c and d we can conclude that the entry is not from the reconnection region:



**Figure 5.** hot spot tracing for (a) particle No.1 trajectory in a background magnetic field from the code, (b) particle No.1 trajectory with the currents from the simulation, (c) particle No.2 trajectory in a background magnetic field from the code and (d) particle No.2 trajectory with the currents from the simulation



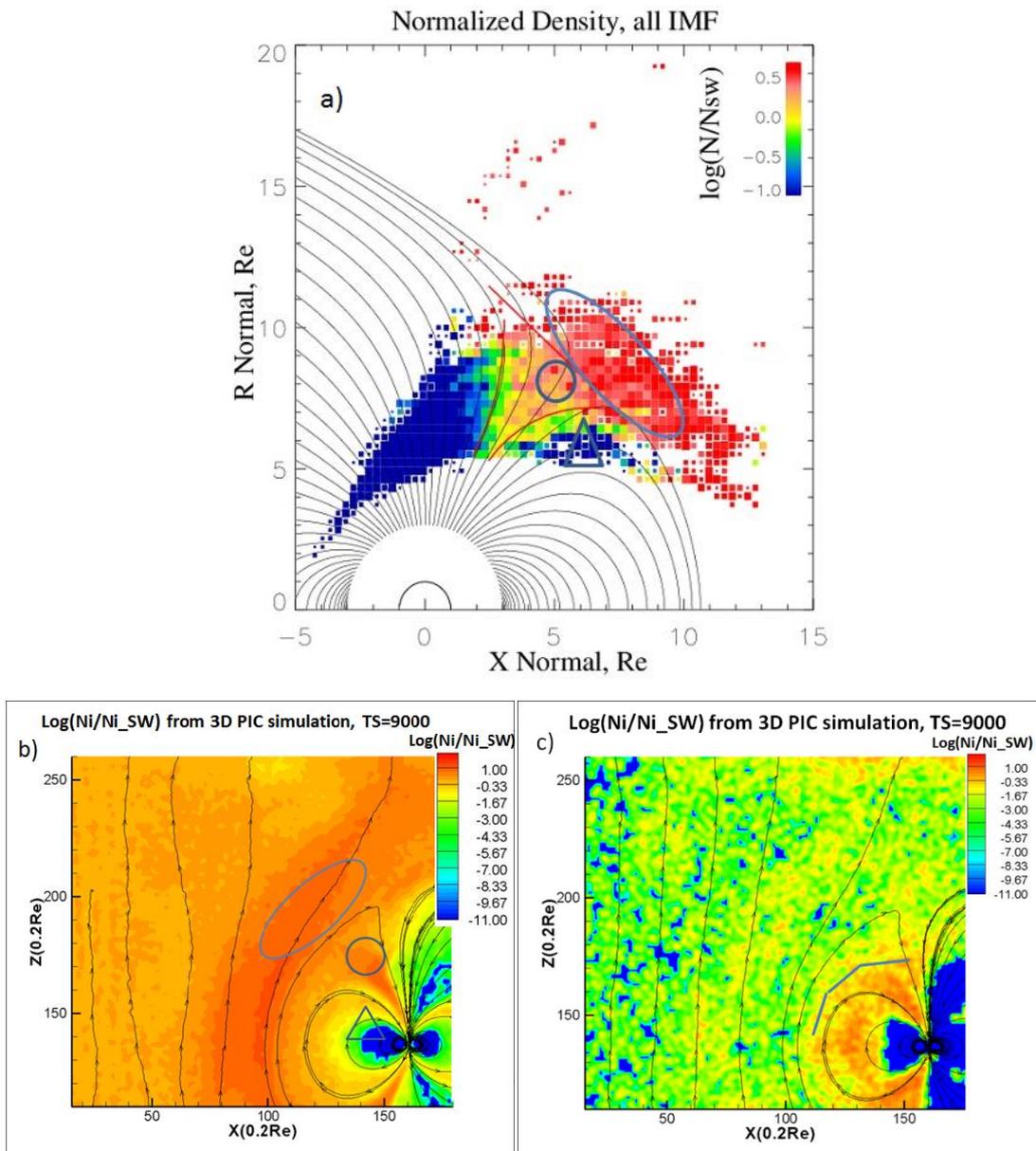
**Figure 6.** particle trajectories for 5 ions under northward IMF condition. The circle shows the cusp particle gate

Also, as it is clear from Fig.6 that is a particle trajectories of 5 ions from the simulation, cusp entry gate for all 5 particles is almost in the same area which is marked with the circle.

### 6. Comparing with Experimental results

The global characteristics of the high-altitude cusp and its surrounding regions are investigated using a three-year statistical survey based on data obtained by the Cluster spacecraft by Lavraud et al. (2004). He reported the density, temperature, kinetic and magnetic pressures and bulk velocity averaged for the different IMF conditions for the high altitude cusp and surrounding

boundaries. As our results shows, ion density results from 3D PIC code are in good agreement with the experimental data from Lavraud et al. (2004). As it is obvious from the Fig. 7a, density is decreasing from the magnetosheath towards the Earth. In our results (Fig. 7b) from PIC simulation the oval is the place of maximum density in the magnetosheath and also the circle is corresponding to the high density in the cusp and triangle is the place of low density near the Earth location. But as the results from the test particle shows in the Fig. 7c we have a structure of high density not from the magnetosheath, but from the marked area in the cusp.



**Figure 7.** The spatial distribution of the ratio of the measured density to that monitored in the solar wind (a) for all IMF conditions (Adapted from Lavraud et al., 2004). (b) from 3D PIC simulation results that used for hot spot tracing and (c) from test particle method.

## 7. Conclusion

To achieve better understanding of results in PIC simulation, scientists usually use the test particles method, which uses fixed magnetic field and a limited number of particles. In this work, we came to this idea to see if we can do the same thing with dynamic magnetic field and self-consistent interactions. Then we saw that we can tag particles in the considered region (hot spot) and go backward in the simulation, trace those particles to visualize particle dynamics on the way to the hot spot. As compared to the test particle method, this method may have a higher cost in terms of computer time required for simulation, but is worthwhile as it gives more exact dynamics of the particles. Here, our initial results, Figs 2 and 5d shows slightly different results from the common accepted physics of particle entry to the cusp from high altitude reconnection site that for example has been done by non-self-consistent test particle method (Walker et al. 2003). However to make this idea clear, it needs more investigation using this method and also analyzing physical quantities including fields and energies that we will try to do in future works.

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