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2D DC resistivity forward modeling based on the integral equation method and a comparison with the RES2DMOD results

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Abstract

A 2D forward modeling code for DC resistivity is developed based on the integral equation (IE) method. Here, a linear relation between model parameters and apparent resistivity values is proposed, although the resistivity modeling is generally a nonlinear problem. Two synthetic cases are considered for the numerical calculations and the results derived from IE code are compared with the RES2DMOD that is a standard software for 2D resistivity forward modeling. For the first synthetic case, a model of resistive block surrounded by a homogenous medium is considered in different depths from 0.5 m to 4 m. For the nearest case to the surface, the IE pseudo-section is similar to its counterpart derived by RES2DMOD but its RMS error is a large value of 13.9 %. Increasing the depth of the anomaly results in decreasing of RMS values to 5.4 % for the deepest case and it is in correspondence with diminishing of the nonlinearity effects of electric fields for larger distances from the sources. The second model is composed of four conductive anomalies embedded in different depths. Visual comparison of IE response with software is indicative of high similarity of them, and RMS error for this relatively complex model is 7.5%, which can be an acceptable misfit for a linear forward operation. A very simple inversion algorithm using linear forward operator is applied on a real data set of a landfill survey in Germany collected by Wenner alfa array to demonstrate its productivity for practical applications. Reconstructed model using IE method is comparable with the inverted model derived by RES2DINV software, and it represents a good similarity with the original model.

Keywords: Forward Modeling, Integral Equation, Resistivity, RES2DMOD**.**

1. Introduction

Forward modeling plays an important role in geophysics, because: 1) one of the main applications of the forward methods in geophysics is their implementation in inversion procedures (Jahandari and Farquharson, 2013), 2) if one is dealing with field campaign, for example resistivity survey, some important questions about choosing the best array, distance between data points, distance between profiles and having insight into some characteristics of anomaly can be achieved by forward modeling; and 3) forward modeling can be used to investigate whether features included in a model obtained by inversion that are constrained (or even required) by the input data (Simpson and Bahr, 2005).

In DC resistivity modeling by numerical methods, a true earth structure is replaced by one for which a numerical approximation to Maxwell's equations can be made and evaluated. The numerical calculation methods for forward modeling of DC resistivity are mainly: integral equations (Dieter et al., 1969; Pratt, 1972; Hohmann, 1975; Lee, 1975; Daniels, 1977; Okabe, 1981; Oppliger, 1984; Xu et al., 1988; Mendez- Delgado et al., 1999), finite element (Coggon, 1971; Fox et al., 1980; Pridmore et al., 1981; Holcombe and Jiracek, 1984; Sasaki, 1994; Tsourlous and Ogilvy, 1999; Li and Spitzer, 2002, 2005; Marescot et al., 2008; Ren and Tang, 2010) and finite difference (Mufti, 1976; Dey and Morrison, 1979; Scribe, 1981; Spitzer, 1995; Zhao and Yedlin, 1996). The finite-difference and finite-element methods are appropriate for arbitrary structures, and are much more flexible than the integral-equation method; however, they are very time-consuming and a very large amount of computer storage is required to solve the large linear equations (Wu et al., 2003). The main limitation of the IE method is that the background conductivity model must have a simple structure to allow for an efficient Green's function calculation (Zhdanov and Michael, 2009). Fortunately, the most widely used

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background models in resistivity explorations are those formed by horizontally homogeneous layers. The theory of Green's functions for layered models is very well-developed and lays the foundation for efficient numerical algorithms. Any deviation from this 1D background model must be treated as an anomalous conductivity. The main advantage of the IE method in comparison with the FD and FE methods is the fast and accurate simulation of the response in models with compact 2D or 3D bodies in a layered background.

Generally, real earth is three dimensional (3D) and its three dimensional modeling is the most accurate way for investigating the subsurface; however, most of the times 2D modeling is also preferred. When the target is infinitely oriented in one direction, which is a very often case, 2D modeling is a good approximation of real earth and it can give us a proper image of the subsurface. The 2D resistivity modeling has been used during last decades for different applications such as: detecting sinkholes (Schoor, 2002; Fehdi et al., 2011), sedimentary rocks (Auken and Christiansen, 2004), for detecting small-scale targets (Candansayar and Başokur, 2001), for detecting buried cavities along with seismic refraction tomography (Cardarelli et al., 2009), to identify sediment-filled faults affecting the construction of a dam near Tecate, Baja California, Mexico (Perez-Flores et al., 2001), for groundwater exploration in a hard rock (Nwankwo, 2011), etc.

In this paper, the 2D forward modeling of DC resistivity utilizing the IE method which was first introduced by Perez- Flores et al. (2001) is used. The method is described briefly, then two numerical examples are introduced and the IE forward results are compared with the results of the RES2DMOD software. Finally, a very simple inversion algorithm by taking the advantage of the linear IE forward operator is applied on a real data set of a landfill in Germany to show the efficiency of the forward mapper for real cases.

2. Methodology

Classical scattering equations derived from Maxwell's equations for frequency of zero are the basis of the DC resistivity modelling using integral equation method. In this method, a given model consists of two parts: background medium and anomalous zone. Background medium is considered as the reference framework, and scattered field produced by anomalous zone is computed as the forward response. Maxwell's equations are nonlinear with respect to the electrical conductivity, and consequently resistivity forward problem is nonlinear. Perez- Flores et al. (2001) made a linear relation between the logarithm of apparent resistivity and logarithm of true resistivity by using a simple linear approximation. The 3D forward formula in integral equation form is:

$$\log \rho_a(r_A.r_B.r_M.r_N) = \frac{C}{4\pi^2} \times \int M(r_A.r_B.r_M.r_N.r') \times \log \rho_a(r')\, d^3 r' \qquad (1)$$

where C is geometrical factor of the array which is $n(n+1)(n+2)a$ for dipole-dipole configuration (a and n are dipole separation and an integer, respectively). Parameters r_A, r_B, r_M, r_N and r' describe the position vectors of electrodes A, B, M, N and anomaly, respectively. M is as (Perez- Flores et al., 2001):

$$M(r_A.r_B.r_M.r_N.r') = L(r_A.r_M.r') - L(r_A.r_N.r') - L(r_B.r_M.r') + L(r_B.r_N.r') \qquad (2)$$

where

$$L(r_i.r_j.r') = \frac{(r' - r_i)(r_j - r')}{|r' - r_i|^3 \left|r_j - r'\right|^3} \quad . \quad i = A\,.B \quad and \quad j = M.N \qquad (3)$$

In fact, integral form of the interested forward problem can be considered as a Fred-Holm Integral Equation of the first kind (IFKs). Integrating from Equation (1) in y direction from -∞ to ∞ leads to the 2D form of IFKs:

$$d(s) = \int G(s.x.z)m(x.z)dxdz \qquad (4)$$

Where s stands for current and potential electrodes, d is forward response, $(x.z)$ are coordinates of points of the interested area, G is kernel and m is the model.

In this case, the subsurface is divided into $n_x \times n_z$ cells and discretizing the previous equation gives rise to the following matrix equation:

$$d = Am \qquad (5)$$

where (Perez- Flores et al., 2001):

$$A = \frac{n(n+1)(n+2)a}{4\pi}(I_{AM} - I_{AN} - I_{BM} + I_{AN}) \tag{6}$$

and

$$I_{AM} = \begin{cases} \frac{4(c^2+p^2)E(q)-8p^2K(q)}{cp^2(c^2-p^2)^2}\left(x_{St} - z'^2 + c^2\right) - \frac{4U}{c^3 q} \ . & for \ \ c > p \ . q = \sqrt{\frac{c^2-p^2}{c^2}} \\ \frac{4(c^2+p^2)E(q)-8c^2K(q)}{pc^2(p^2-c^2)^2}\left(x_{St} - z'^2 + p^2\right) - \frac{4U}{p^3 c} \ . & for \ \ p > c \ . q = p\sqrt{\frac{p^2-c^2}{p^2}} \\ \frac{\pi}{4}\left[\frac{1}{c^3} - \frac{1}{c^5}\left(x_{St} - z'^2\right)\right] . & for \ \ c = p \end{cases} \tag{7}$$

and

$$c^2 = (x' - x_S)^2 + z'^2 \quad . \quad p^2 = (x - x')^2 + z'^2$$
$$U = \frac{E(q)}{q(1-q^2)} - \frac{E(q)}{q} \quad . \quad x_{St} = (x' - x_S)(x - x') \tag{8}$$

x_S and x stand for x coordinates of current and potential electrodes, respectively. x' and z' indicate the coordinates of cell's centers, and K(q) and E(q) are in turn complete elliptical integral of the first and second kind. Matrix A and the column vector m are forward operator and model parameters, respectively, and Equation (5) represents the forward problem.

3. Numerical results

Two numerical examples are employed to investigate the efficiency of the linear IE forward modeling. The models are compared with the models resulted from the standard RES2DMOD software both qualitatively and quantitatively. It should be mentioned that dipole-dipole array is used for two forward numerical cases, but this method can be used for any DC resistivity array and as Wenner alfa array is manipulated for the real case.

3-1. Resistive block in a homogeneous medium

The first numerical example consists of a 100 Ω.m resistive block in a homogenous background with the resistivity of 10 Ω.m. Anomalous body has a depth of burial of 0.5 m and its horizontal and vertical extensions are 3 and 2 m, respectively (Figure 1). Forward responses (pseudo-sections) derived from IE method and RES2DMOD software, and their difference with pseudo-sections can be observed in Figures 2, 3 and 4, respectively. Visual comparison of them is indicative of their good likeness, but RMS error for IE method relative to the software result is 13.9 % that is a large error from quantitative point of view, but it should be considered that the anomaly is near to surface (source positions) and an approximate technique is utilized. If we increase the depth of the anomaly to 1, 2, 3 and 4 m and calculate their corresponding RMS errors, it can be seen that RMS errors have a decreasing trend to 11, 8, 6.3 and 5.4 % (Table 1), respectively, which is in agreement with reducing the nonlinearity manner of electric field by moving to larger distances from the source or sources. In other words, electric field behavior approaches to linearity at large distances from it and therefore the linear forward operator can be a good approximation of the nonlinear behavior of the problem.

Table 1. RMS error for different resistive block depths.

Depth to top of anomaly (m)	0.5	1	2	3	4
RMS error (%)	13.9	11	8	6.3	5.4

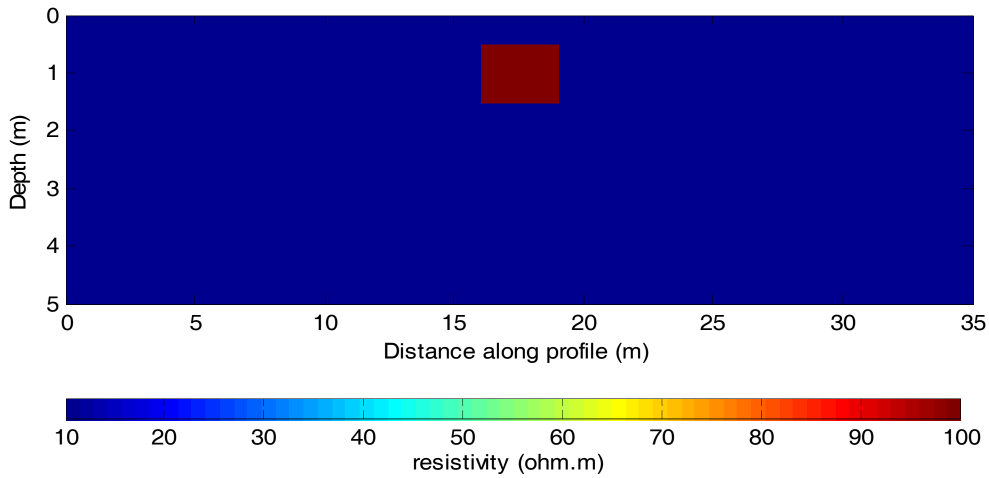

Figure 1. Model of a resistive block surrounded by a homogeneous medium that is a simple model for DC resistivity method.

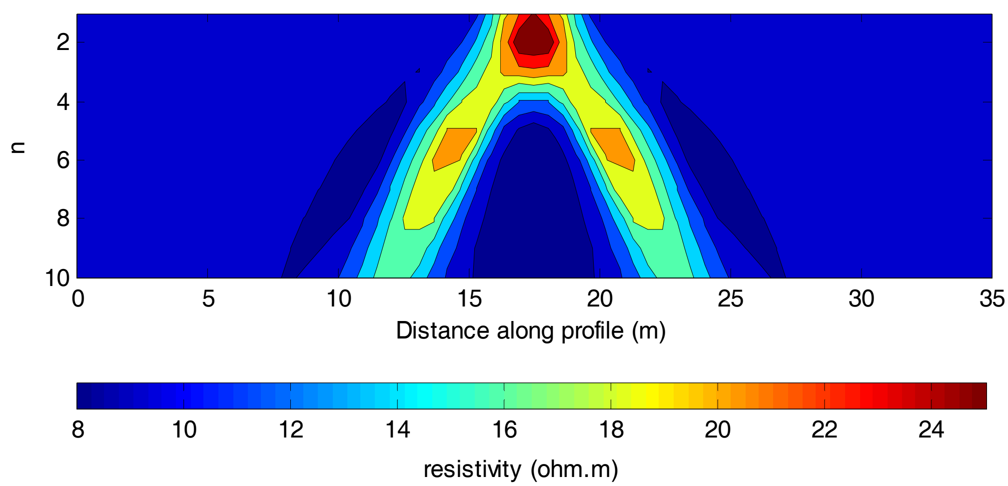

Figure 2. Pseudo-section derived from IE method when depth to top of the block is 0.5 m. Data sampling interval and dipole separation both were 1 m. Symmetry can be observed from this pseudo- section.

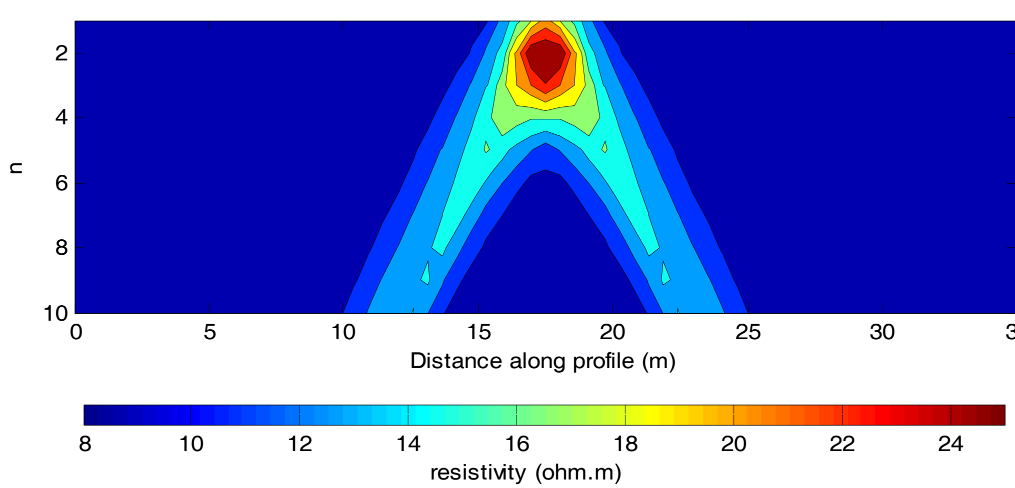

Figure 3. Pseudo-section derived from RES2DMOD software when depth to top of the block is 0.5 m. Data sampling interval and dipole separation both were 1 m. As the result of IE, it also shows a symmetry.

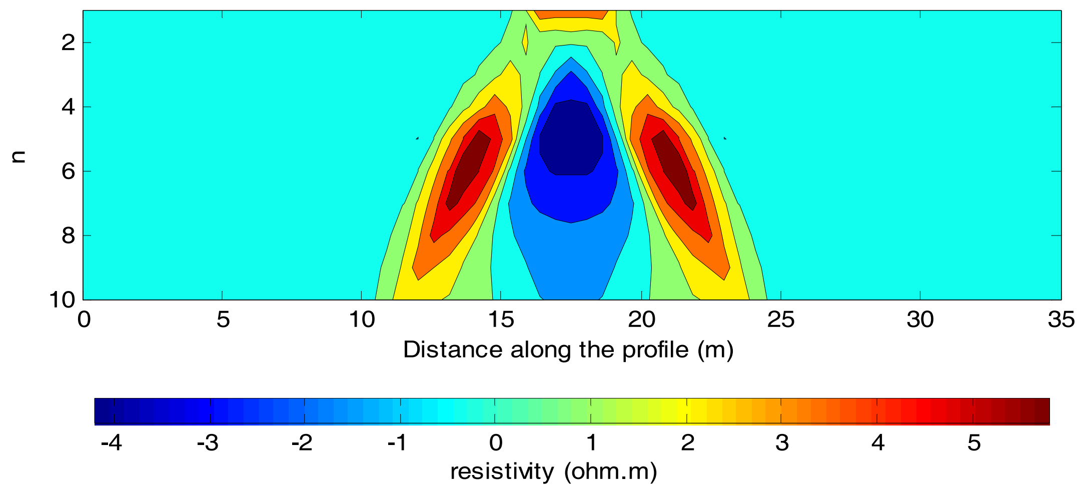

Figure 4. Difference of pseudo-sections for the block with its depth to top equal to 0.5 m.

3-2. Complex model

For the second synthetic case, a model consisting of four conductors of 20 Ω.m were considered with different dimensions in a homogenous medium of 100 Ω.m (Figure 5). Data sampling interval and dipole separation were chosen to be equal to 10 m. In general, the resistivity responses of single bodies are somewhat complex and they incline to be mixed with those of the nearby conductors. Due to the interfering effect of the shallower anomalies, existence of the deep conductor is difficult to be recognized. Figure 6 portrays the pseudo-section calculated by IE code while pseudo-section derived by RES2DMOD software and difference between two pseudo-sections are represented in Figures 7 and 8, respectively. The shape of the pseudo-section is very similar to RES2DMOD result, but there is a difference between their values. Calculated RMS error is 7.5% that can be an acceptable error and it can be said that this synthetic model showed us the reliability of the linear IE forward operator even for complex models. This linear IE forward mapper allows us to have a linear inverse problem for which there are many techniques to be used for solving it.

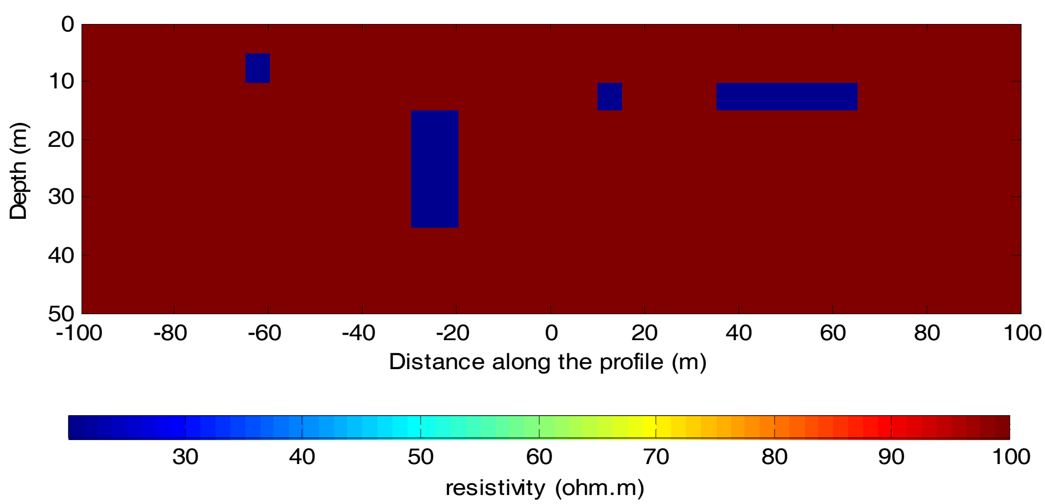

Figure 5. Four conductive bodies immersed in a resistive host medium that can be considered as a complex resistivity model.

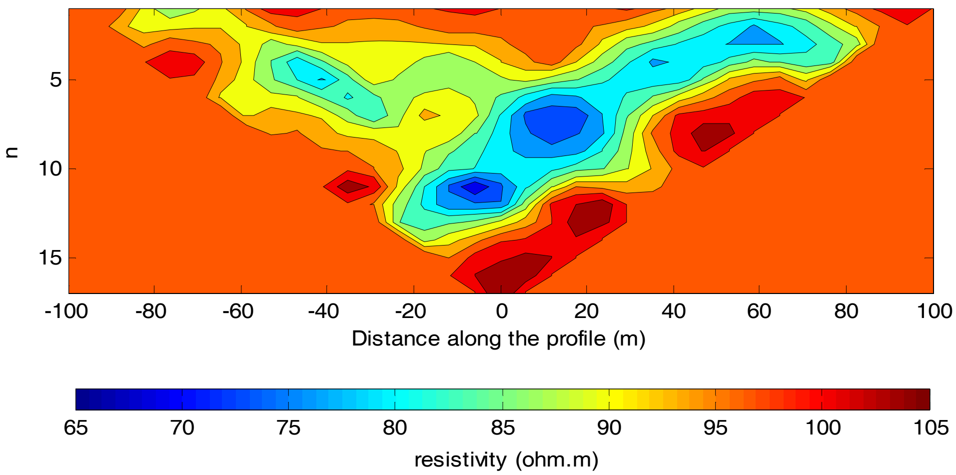

Figure 6. Pseudo-section calculated by IE method. It is difficult to find a single block response and they incline to be mixed with nearby anomalies.

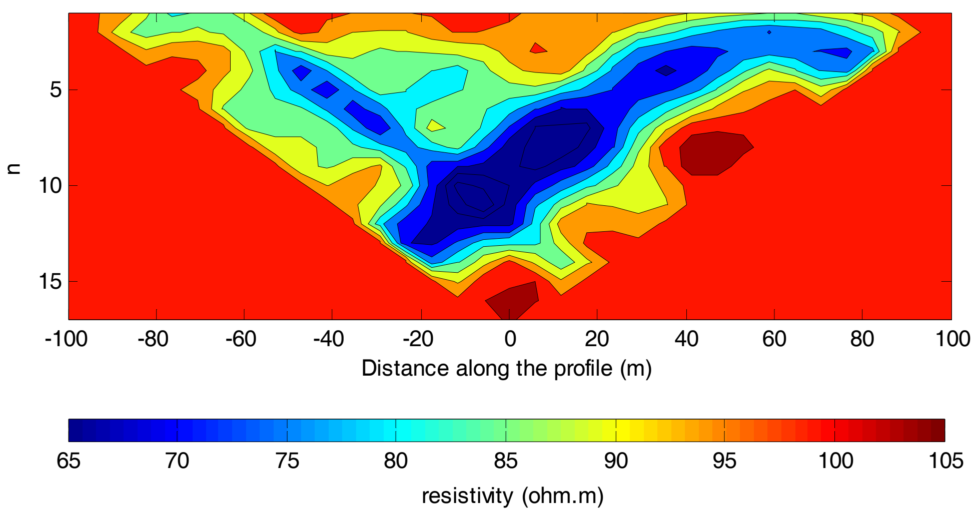

Figure 7. Pseudo-section obtained by RES2DMOD software. Comparing this standard result with IE one is expressive of the productivity of IE method.

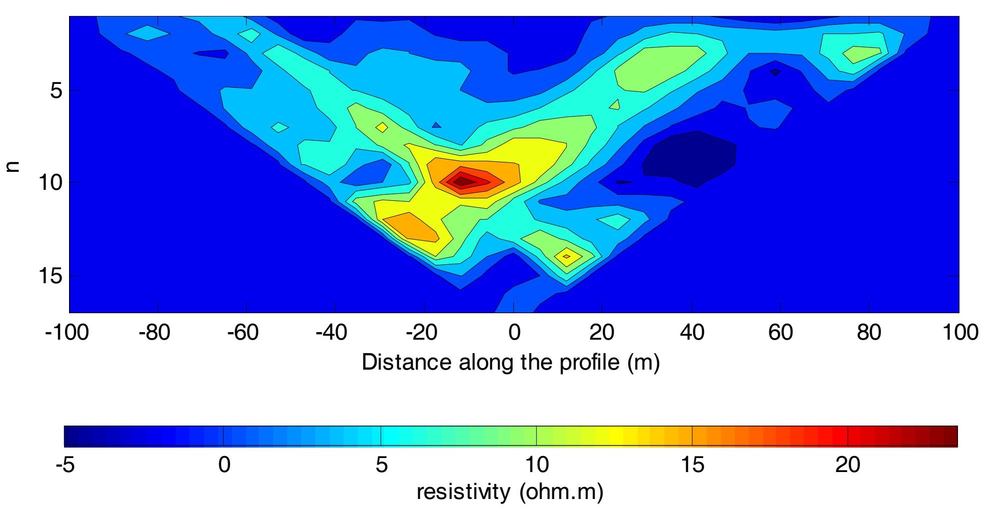

Figure 8. Difference of two pseudo-sections. For n=10 and approximately in the middle of the array, the largest error is occurred.

3-3. Real case

In order to prove the efficiency of the IE forward operator, a very simple inversion algorithm is applied on a real data set of a landfill in Germany. It should be accentuated that the inversion of resistivity data is not interested here, and it is beyond the scope of the paper. Recovering process of the true model is made utilizing the following simple inversion formula, which is well known as regularized minimum length solution with respect to an initial model:

$$m = m_a + A^T(AA^T + \alpha^2 I)^{-1}(d - Am_a) \qquad (9)$$

m_a and α are initial model and regularization parameter, respectively and superscript T means transpose operation. Inversion is an iterative process and starts with a homogenous model, then initial solution is updated during each iteration by equalizing m_a with m.

3-4. Field data (Wenner array)

True model of a landfill in Germany is shown in Figure 9, which can be found in the RES2DMOD software. The data were collected using Wenner alfa array for two different electrode separations of 3 and 6 m, and the pseudo-section derived by the software is represented in Figure 10. Retrieved model from the data by RES2DINV can be seen in Figure 11 with MATLAB display, while Figure 12 shows the inverted model from the IE method. It should be noted that RES2DINV default display for inversion result has a V-shaped representation and it also allows us to have rectangular demonstration; however, it is not recommended for Wenner array due to its low model sensitivity values near edges. Figures 11 and 12 tell us that the IE inversion result is comparable with reconstructed model by RES2DINV, and it can be asserted that it shows a high resemblance with the original model. Therefore, the productivity of the IE forward operator was made obvious by introducing this real data set. It should be mentioned that this satisfying result was obtained for landfill anomaly using linear IE method in spite of its specialty for recovering compact anomalies in homogeneous or layered backgrounds.

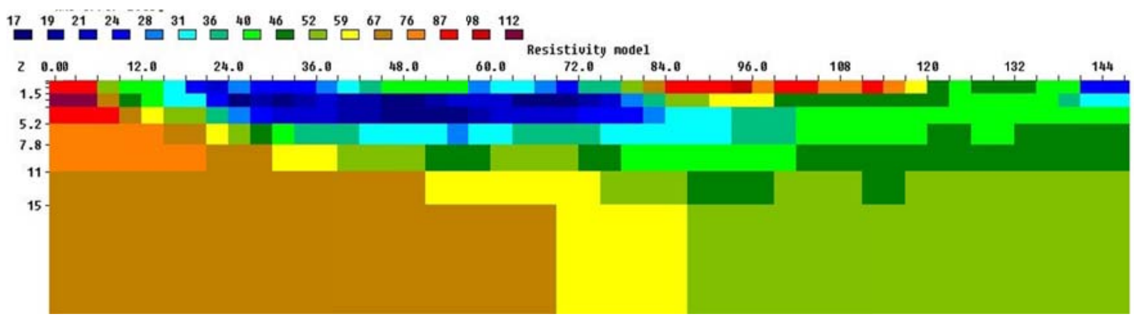

Figure 9. True model of the landfill that can be found between default models of RES2DMOD software.

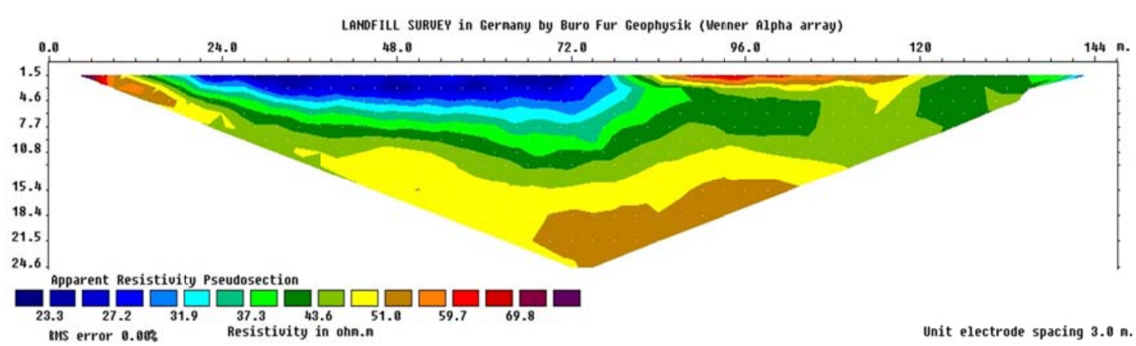

Figure 10. Pseudo-section of the landfill model. Two dipole separations were used: 3 m with n from 1 to 8 and 6 m with n from 5 to 8. This representation of the RES2DMOD software is just for dipole separation of 3 m.

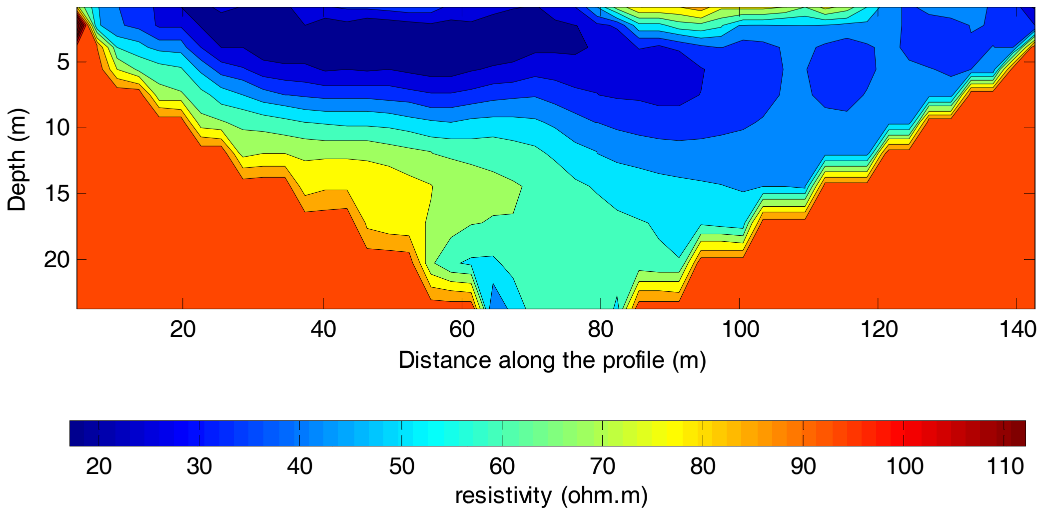

Figure 11. Recovered model by RES2DINV software with the RMS error of 2.3 %. Reconstructed model is in a good agreement with the true model.

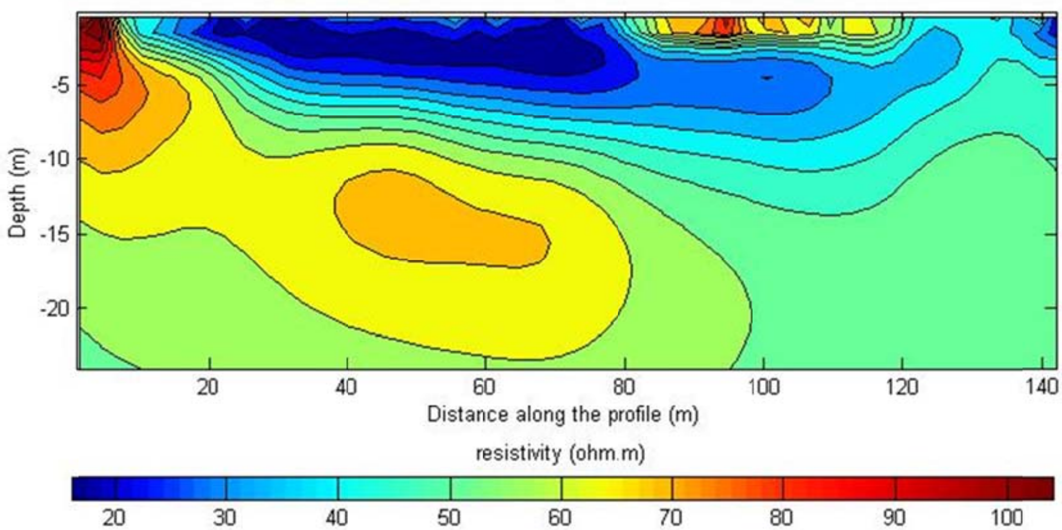

Figure 12. Inversed model by applying the IE code on landfill data with the RMS error of 4.5%.

4. Conclusions

A 2D forward modeling code for DC resistivity was written based on the integral equation method, and the results of the code were compared with RES2DMOD software through two numerical examples: I) At first, model of a resistive block immersed in a homogenous background was considered, II) a complex model of four conductive bodies in a homogenous background was assumed for which two blocks have the same dimensions but with different depths and two others had different dimensions and depth ranges. Visual comparison of the IE result with RES2DMOD forward response for resistive block shows a good agreement between them, but when the block is very close to the surface, the RMS error is a large value and augmenting its depth leads to decreasing the error so that for the block with depth of 4 m, RMS error is 5.4%. Decreasing RMS error due to the depth increase means better performance of the linear IE forward mapper and this is congruous with declination of nonlinearity effects of electric fields in larger distances from the source. In addition to the good resemblance with the RES2DMOD pseudo-section for the complex model, IE forward modeling has a relatively acceptable RMS error of 7.5%. A landfill real data set, collected by Wenner alfa array in Germany, was used to show the effectiveness of the linear IE forward mapper through applying a very simple inversion algorithm on the real data. Comparing the model of IE code with the model of RES2DINV inversion as well as the original model is clearly demonstrative of its usefulness for practical cases.

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