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Multi-Observations Initial Orbit Determination based on Angle-Only Measurements

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Abstract

A new approach with the ability to use the multiple observations based on the least square approach has been proposed for initial orbit determination. This approach considers the Earth's Oblateness by using the developed Lagrange coefficients. The efficiency of the proposed method has been tested in two scenarios. The first scenario is to use the simulated and the second one is to utilize the real angle-only observations for the GRACE-like and GPS-like satellites. Under the first scenario, the ground-based observations are produced using the reduced-dynamic orbit generated by GFZ. Then, various error levels were added to the produced azimuth and elevation observations. The results show that considering the Earth's oblateness could improve the accuracy of the initial orbit determination by six times for a GRACE-like satellite, and by 60 times for a GPS-like satellite. Afterward, under the second scenario, the real observations of the SLR station were used. In view of increasing in the number of observation tests, by increasing the numbers of the observations from 3 to 15, the accuracy of initial orbit determination was improved from 1496 to 8 m using the SLR data for the GRACE-A satellite.

Keywords: Initial Orbit Determination, Ground-based observations, Celestial Mechanics, Least Square Approach.

1. Introduction

Traditionally, Initial Orbit Determination (IOD) has been used for the planets and asteroids orbit determination in celestial mechanics. In the early years, the planet orbit was determined using the minimum needed observations made at ground-track stations. The initial orbit determination means the process of estimating Keplerian elements of planets, asteroids, satellites, debris, etc. using the ground-based observations (Montenbruck and Gill, 2000). The azimuth and elevation of a celestial target at separated times have traditionally been observed as ground-based observations. Then, the angle-only methods have been regarded as a backbone of the initial orbit determination from the past up to now, not only for planets, asteroids and debris, but also for satellites. Nowadays although, Global Navigation Satellite Systems (GNSS) play a key role in the precise orbit determination of satellites (Cerri et al., 2010; Jäggi et al., 2007; Van Helleputte and Visser, 2008), it is too difficult to ignore the ground-based methods especially the optical ones. The major cause of this fact is the passive nature of the angle-only methods. The advantage of the angle-only methods could be very vital for the orbit determination of the unknown or newly launched satellites besides debris orbit determination and space surveys (Milani et al., 2008).

After Tycho Brahe's attempts to develop superior observational techniques to study celestial objects (McCutcheon and McCutcheon, 2005), the first method of finding the orbit of a celestial body from three observations was devised by Newton which was given in the Principia in 1678 (Bate et al., 2013). The fundamental works on the initial orbit determination methods were published by Lagrange in 1778 and others were published as: Vallado (2001), Laplace (1780) and Gauss (2004). Gauss's method is a simple method which uses the right ascension and declination at three observation times to determine natural and artificial celestial bodies (Vallado, 2001). The Gauss exact method uses the exact functions of the Lagrange coefficients (f and

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g) instead of a truncated series form of the f and g functions in an iterative algorithm (Curtis, 2005). Escobal (1965) developed another method named Double-R, a newer method presented by Gooding (1996) that could be classified as the angle-only method for IOD using the angle-only observations. These methods have been reviewed and compared by some authors (Celletti and Pinzari, 2006; Gronchi, 2009; Merton, 1925; Milani and Gronchi, 2010; Taff, 1984).

In addition to classical efforts for initial orbit determinations, some literatures have been published on this issue. A theoretical and numerical comparison was made between different procedures by (Celletti and Pinzari, 2005). Farnocchia et al., (2010) proposed two algorithms for debris and common orbits to provide a full preliminary orbit of an Earth-orbiting object with a number of observations lower than the classical methods. Some literatures have focused on the definition of an orbit determination of space debris and the related admissible region (Farnocchia et al., 2010; Tommei, et al., 2007; Doscher, 2018). The initial asteroid orbits were determined by the least squares adjustment of an arbitrary number (N) of optical and radar observations by Kristensen (2007). Following that, a few articles mentioned the IOD method using the multiple data (Karimi and Mortari, 2011, 2013; Kristensen, 2009). Hu et al. (2019) investigated the use of space-based tracking data to determine the initial orbit of low-Earth orbit target satellites.

Although extensive research has been carried out on the initial orbit determination using different observations, almost all of the previously mentioned methods suffer from some serious drawbacks. The classic IOD methods are highly dependent on the type of observations, and the method should be different when other types of observations are measured. Another problem with these approaches is that they fail to take further observations into account. As an example, three sets of observed azimuths and elevations are used to solve IOD in the angle-only approach and more observations are unusable. Due to further limitations, such explanations tend to overlook the fact that the satellites are affected by perturbing forces. The main hypothesis about almost all of them is that only the force acting on a satellite is central force, i.e. Keplerian motion. Although the hypothesis of the Keplerian motion could be sufficient for planets and asteroids IOD because they are considerably high orbiter, it could not be acceptable for satellites, especially LEO ones.

The analysis undertaken in this paper attempts to bridge these gaps in the literature by proposing a new algorithm for IOD. In this paper, it is tried to present a methodology based on Least Squares (LS) estimation for Initial Orbit Determination. The proposed method is not limited in using more observations. In addition, in order to increase the accuracy of IOD, the Earth oblateness is considered. For this propose, the developed Lagrange coefficients were used in IOD. The Lagrange coefficients were developed by Lin and Xin (2003) by taking into account Earth's oblateness and it was continued by Sharifi and Seif (2011). The proposed method is flexible and practical. It is flexible because the existing knowledge of LS theory can readily be applied in IOD problems and also it can be utilized for a variety of applications in this field. Moreover, it is practical because it is a general methodology that can easily be used in IOD from not only angles-only observations, but also range, range-rate, Doppler, etc., measurements. However, in this paper, as an example and a proof-of-concept, this methodology is applied to angle-only measurements. The latter characteristic of the presented methodology makes it a viable alternative to conventional methods that are not usually able to deal with all data types in a unified way.

Section 2 begins by laying out the theoretical dimensions of the research, and the formulation of the new proposed method is described in more details. In the following section, the proposed method has been validated by two scenarios. The first one is to do with the use of the simulated observations for IOD and the second one deals with the real observations obtained from Satellite Laser Ranging (SLR) stations.

2. Mathematical modeling of the multi observations

In this paper, it is hypothesized that the multi observations measured from a ground station can be used to solve IOD problem. The authors have proposed a new method that is independent of the number of observations. It could be independent of the type of the observations (range, range rate, Doppler shift, etc.) too. In contrast to the classical methods, which are carried out in the central field, the Earth's oblateness has been considered in the new proposed method.

In our proposed method, $N = 2n+1$ sets of the angle-only observations are used to compute the best estimation of the position and velocity vector at the middle point t_n. For this purpose, at first, the observation vector should be formulated as a function of the position and velocity vectors (state vector) at t_n, $\underline{r}_n$ and $\underline{\dot{r}}_n$.

$$\underline{l} = \underline{f}(\underline{r}_n, \underline{\dot{r}}_n) \quad (1)$$

Moreover, the state vector of the target at the middle point is directly inestimable due to the nonlinearity of the system of equations. Then, these equations should be linearized and the problem should be solved using an iterative scheme. The linearization has been carried out around the initial value of the position and velocity vectors at t_n. Assume the sought-after correction is so small that the linearization can yield the accurate approximation of the equations. The linearized form of Eq. (1) could be computed from:

$$\underline{l} = \frac{\partial \underline{f}}{\partial[\underline{r}_n, \underline{\dot{r}}_n]}\left(\begin{bmatrix}\underline{r}_n \\ \underline{\dot{r}}_n\end{bmatrix} - \begin{bmatrix}\underline{r}_n^0 \\ \underline{\dot{r}}_n^0\end{bmatrix}\right) + f(\underline{r}_n^0, \underline{\dot{r}}_n^0) \quad (2)$$

The solution process is started with an initial guess of the position and velocity vector. Numerically, the problem can be expressed as an optimization problem. The aim is to find the correction to the initial state vector in a way that the deviation of the computed azimuth and elevation using the estimated position vectors of space target with respect to given sets of the angle-only observations is minimized.

The Eq. (2) is equivalent to:

$$\begin{cases} \underline{d} = \underline{A}\begin{bmatrix}\delta\underline{r}_n \\ \delta\underline{\dot{r}}_n\end{bmatrix} - \underline{d\ell} \\ \|\underline{d}\| \quad \rightarrow \quad \min \end{cases} \quad (3)$$

with the misfit vector $\underline{d}$, $\|\underline{d}\|$ the norm of the misfit vector, the design matrix $\underline{A}$ and the observation misclosure vector $\underline{dl} = \left[\underline{l}_i - \underline{l}_i^0\right]$.

Applying the method of least squares yields:

$$\underline{d\hat{s}} = (\underline{A}^T\, \underline{P}\, \underline{A})^{-1} \underline{A}^T\, \underline{P}\ \underline{dl} \quad (4)$$

where P is the weight matrix of the observations. For the ease of implementation, it is the set to be equal to the identity matrix. The procedure has schematically been summarized in Figure 1.

Based on the procedure presented in Figure 1, at first, the initial estimate of the position vector at the middle point was obtained by using the exact Gauss method. The observation sets at the first, middle and final points were selected for the computation of the position vector of the target at the middle point $\underline{r}_n^0$ because of the maximum stability. The initial estimate of the velocity vector at the middle point $\underline{\dot{r}}_n^0$ was computed using Laplace method. The initial value should iteratively be improved so that the computed azimuth and elevation can optimally be matched to the observed ones at every point. For evaluating computed observations, the position vectors of the target at $t_1, t_2, \ldots, t_{2n+1}$ are required. However, the initial guess of the position and velocity vector has been carried out only for the middle point. Then, a propagator is necessary to bridge the gap. The initial guess of the state vector at the middle point, $\underline{r}_n^0$ and $\underline{\dot{r}}_n^0$, could be propagated to other points using generalized Lagrange coefficients in the J2 field, the gravity field of the Earth by considering the Earth's oblateness (Sharifi and Seif, 2011):

$$\underline{r}_i = \underline{F}(t_i)\underline{r}_n(t_n) + \underline{G}(t_i)\underline{\dot{r}}_n(t_n) \quad (5)$$

where $\underline{F}(t_i)$ and $\underline{G}(t_i)$ are Lagrange matrices.

Taylor expansion coefficients were derived for a J2 field by Sharifi and Seif (2011). They are presented in Appendix A, too.

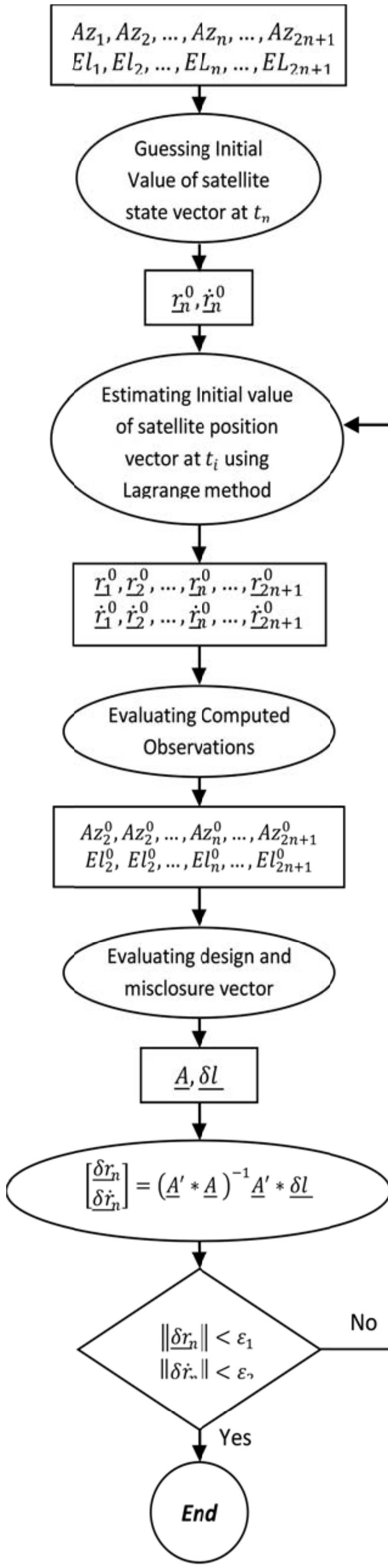

Figure 1. The algorithm of proposed method.

By the known position vector of the ground station and the initial value of the satellite position vector at t_i, the computed observations can be obtained via:

$$\underline{l}_i^0 = \begin{bmatrix} Az_i^0 \\ El_i^0 \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{\rho_E^0}{\rho_N^0}\right) \\ \sin^{-1}\left(\frac{\rho_Z^0}{\sqrt{\rho_E^{0\,2} + \rho_N^{0\,2} + \rho_Z^{0\,2}}}\right) \end{bmatrix}_i \tag{6}$$

where

$$\begin{bmatrix} \rho_E^0 \\ \rho_N^0 \\ \rho_Z^0 \end{bmatrix}_i = \underline{J}' \, \underline{T}(t_i) \, (\underline{r}_i^0 - \underline{R}_i), \quad i = 1, 2, \ldots, 2n+1 \tag{7}$$

where $\underline{R}_i$ is the position vector of the ground station. The Jacobian Matrix $\underline{J}$ is given by:

$$\underline{J} = \begin{bmatrix} -\sin(\lambda) & -\sin(\phi)\cos(\lambda) & \cos(\phi)\cos(\lambda) \\ \cos(\lambda) & -\sin(\phi)\sin(\lambda) & \cos(\phi)\sin(\lambda) \\ 0 & \cos(\phi) & \sin(\phi) \end{bmatrix} \tag{8}$$

where $\underline{T}(t)$ is the transformation matrix that transforms the position vector from the Earth-centered Inertial (ECI) into the Earth-centered Earth-fix (ECEF). For more details about transformation matrix, see McCarthy and Petit (2003).

To complete the computation procedure, the design matrix should be calculated. The design matrix $\underline{A}$ is expressed as a product of the partial derivative using the chain rule as:

$$\underline{A} = \begin{bmatrix} \frac{\partial \underline{l}_1}{\partial[\underline{r}_n, \underline{\dot{r}}_n]} \\ \frac{\partial \underline{l}_2}{\partial[\underline{r}_n, \underline{\dot{r}}_n]} \\ \vdots \\ \frac{\partial \underline{l}_n}{\partial[\underline{r}_n, \underline{\dot{r}}_n]} \\ \vdots \\ \frac{\partial \underline{l}_{2n+1}}{\partial[\underline{r}_n, \underline{\dot{r}}_n]} \end{bmatrix}_{2N\times 6} \tag{9}$$

The partial derivatives of the observations (Azimuth and Elevation) with respect to the state vector are:

$$\frac{\partial \underline{l}_i}{\partial[\underline{r}_n, \underline{\dot{r}}_n]} = \frac{\partial \underline{l}_i}{\partial \underline{r}_n}\frac{\partial \underline{r}_i}{\partial \underline{r}_n} = \frac{\partial \underline{l}_i}{\partial \underline{\rho}_i^{ENZ}} \frac{\partial \underline{\rho}_i^{ENZ}}{\partial \underline{\rho}_i^{ECI}} \frac{\partial \underline{\rho}_i^{ECI}}{\partial \underline{r}_i} \frac{\partial \underline{r}_i}{\partial[\underline{r}_n, \underline{\dot{r}}_n]}, \quad i = 1, 2, \ldots, 2n+1 \tag{10}$$

where $\underline{l}_i = [Az_i, El_i]$ is the ground-based observations, the azimuth, and elevation, measured from the ground station at t_i, $\underline{\rho}_i^{ECI}$ is the measured line-of-sight vector in the geocentric system that could be obtained via:

$$\underline{\rho}_i^{ECI} = \underline{T}(t_i)' * \underline{J} * \underline{\rho}_i^{ENZ} \tag{11}$$

where $\underline{\rho}_i^{ENZ}$ is the measured line-of-sight in the topocentric system.

$$\underline{\rho}_i^{ENZ} = \begin{bmatrix} \rho_E \\ \rho_N \\ \rho_Z \end{bmatrix}_i = \rho_i \begin{bmatrix} \cos(El_i) siz(Az_i) \\ \cos(El_i)\cos(Az_i) \\ \sin(El_i) \end{bmatrix} \tag{12}$$

The partial derivatives of the observations with respect to $\underline{\rho}^{ENZ}$ are:

$$\frac{\partial \underline{l}_i}{\partial \underline{\rho}_i^{ENZ}} = \begin{bmatrix} \frac{\rho_N}{\rho_N^2 + \rho_E^2} & \frac{-\rho_E}{\rho_N^2 + \rho_E^2} & 0 \\ -\frac{\rho_E \rho_Z}{\rho^2\sqrt{\rho^2 - \rho_Z^2}} & -\frac{\rho_N \rho_Z}{\rho^2\sqrt{\rho^2 - \rho_Z^2}} & \frac{\rho^2 - \rho_Z^2}{\rho^2\sqrt{\rho^2 - \rho_Z^2}} \end{bmatrix} \tag{13}$$

It is obvious from the fundamental equation of IOD from the ground-based observation $\underline{r}_i = \underline{R}_i + \underline{\rho}_i^{ECI}$ that $\dfrac{\partial \underline{\rho}_i^{ECI}}{\partial \underline{r}_i}$ is equal to identify matrix. $\dfrac{\partial \underline{r}_i}{\partial [\underline{r}_n, \underline{\dot{r}}_n]}$ is half part of the transition matrix between t_i and t_n defined as:

$$\underline{\Phi}(t_i, t_n) = \begin{bmatrix} \dfrac{\partial \underline{r}_i}{\partial \underline{r}_n} & \dfrac{\partial \underline{r}_i}{\partial \underline{\dot{r}}_n} \\ \dfrac{\partial \underline{\dot{r}}_i}{\partial \underline{r}_n} & \dfrac{\partial \underline{\dot{r}}_i}{\partial \underline{\dot{r}}_n} \end{bmatrix} = \begin{bmatrix} \underline{\Phi}_{11} & \underline{\Phi}_{12} \\ \underline{\Phi}_{21} & \underline{\Phi}_{22} \end{bmatrix} \tag{14}$$

Finally, Eq. (15) could be summarized as:

$$\frac{\partial \underline{l}_i}{\partial [\underline{r}_n, \underline{\dot{r}}_n]} = \begin{bmatrix} \dfrac{1}{\rho_N \left(1 + \dfrac{\rho_E^2}{\rho_N^2}\right)} & \dfrac{-\rho_E}{\rho_N^2 \left(1 + \dfrac{\rho_E^2}{\rho_N^2}\right)} & 0 \\ -\dfrac{\rho_E \rho_Z}{\rho^3 \sqrt{1 - \dfrac{\rho_Z^2}{\rho^2}}} & -\dfrac{\rho_N \rho_Z}{\rho^3 \sqrt{1 - \dfrac{\rho_Z^2}{\rho^2}}} & \dfrac{\rho^2 - \rho_Z^2}{\rho^3 \sqrt{1 - \dfrac{\rho_Z^2}{\rho^2}}} \end{bmatrix} * \underline{T}(t_i) * \underline{J}' * \left[\underline{\Phi}_{11} \quad \underline{\Phi}_{12}\right] \tag{15}$$

3. Numerical Analysis

In this section, the efficiency and improvement of the proposed method have been compared with the exact Gauss method. The comparison has been carried out in two scenarios. The first scenario deals with IOD using the simulated observation, and the second scenario uses the real observations obtained from SLR stations. The precise orbit of the GRACE and GPS satellites has been considered as a true orbit to check the obtained results in the simulation scenario. The accuracy of the precise orbit is claimed to be at the centimeter level. In the simulation procedure, the ground-based observations of the satellite (Azimuth, Elevation, and Range) is produced from the known position vector of the ground station and satellite. Various error levels were added to the produced azimuth and elevation observations for simulating a real situation. The satellite position vector was estimated using the Gauss exact and proposed method and compared with the true orbit. In the second scenario, the satellite position vector was estimated using the real angle-only observations. Then, the observed range was compared with the estimated range for checking the efficiency of the method. Furthermore, the improvements given from considering the J2 effect and using multi-observations have been assessed in two scenarios. As said before, in the first scenario, the simulated observations have been used for estimating satellite state vector. The algorithm of the procedure has been shown in Figure 2.

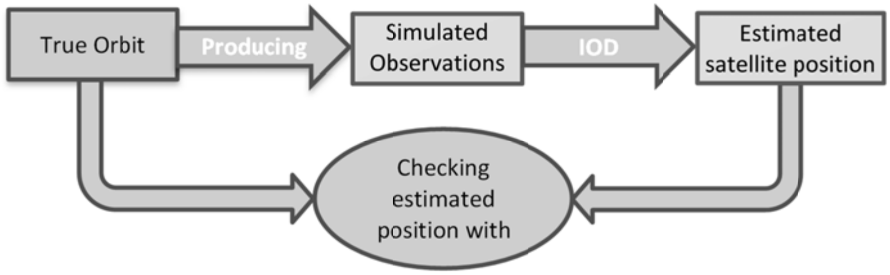

Figure 2. The procedure of the IOD using simulated observations.

Based on the first scenario described in Figure 2, the precise orbit of the GRACE satellite has been converted to ground-based observations. After adding the error to the simulated observation at different levels, the IOD process is carried out using the proposed method. Figure 3 describes the error of the proposed method versus the Gauss approach.

As shown in Figure 3, the proposed method will be ten times more accurate than the Gauss method for a GRACE-like satellite should the accurate observations be used. For inaccurate observations (the error levels larger than 10^{-2}), there is no significant difference between the proposed and Gauss methods. The main reason is that the inaccurate observations do not need a more perfect model that considering the Earth oblateness responsible for the initial orbit improvement obtained in the proposed method. This comparison was repeated for an MEO satellite, GPS satellite, and the result was presented in Figure 4.

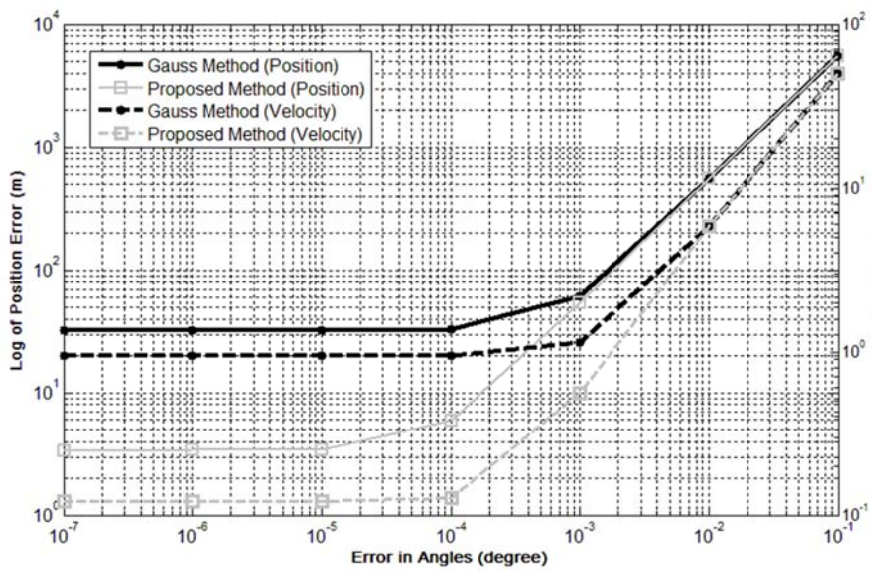

Figure 3. The position (solid lines) and velocity (dashed lines) errors of Gauss and proposed methods on account of using the simulated noisy observations for GRACE satellite.

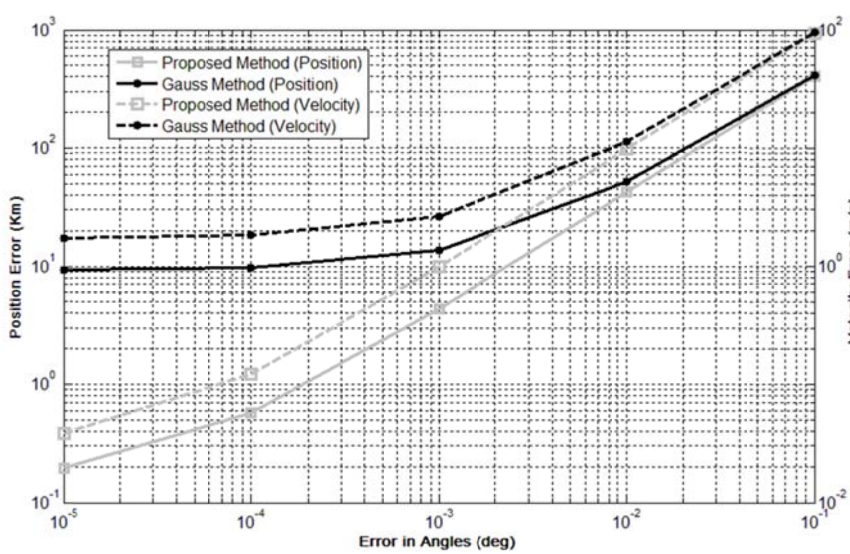

Figure 4. The position (solid lines) and velocity (dashed lines) errors of Gauss and proposed methods on account of using the simulated noisy observations for a GPS satellite.

As shown in Figure 4, the improvement of the proposed method for the MEO satellite is greater than that of the LEO satellite because the time interval between angle observations could be chosen larger. The larger the time interval is, the greater the impact of the Earth oblateness will be. The proposed method is about sixty times more accurate than the Gauss method for a GPS-like satellite if the accurate observations would be used. Like the LEO satellite case, for inaccurate observations (the error levels larger than 10^{-2}), there is no significant difference between the proposed and Gauss method.

In addition to the position and velocity error, we are interested in assessing the error of Keplerian elements. Figure 5 shows the errors of the Keplerian elements (Semi-major Axis, Eccentricity, Inclination, Right Ascension of Ascending Node (RAAN) and Arguments of Latitude) for an MEO-like satellite.

So far, the proposed method has been validated for the least observations needed for IOD. Afterward, the ability of the proposed method to add more observations (more than three sets of azimuth and elevation) was tested. Obviously by adding further observations, the accuracy of the initial orbit determination could be increased. However, the classic method, e.g. the Gauss method, could not use more observations, and it is one of the main disadvantages of these approaches. By proposing a different algorithm, the proposed method attempts to use more observations made in the ground station. The orbit improvement using multi-observations has been described in Figure 6 for the simulated observation. These observations were produced from the real orbit of the GPS-02 satellite determined by GFZ institute. The simulated observations were noised by adding different error levels. The maximum number of observations used in this scenario was 15 and the time interval between two consecutive sets of observations was 5 min. In the procedure of increasing the number of observations, at each step, two sets of observations in both sides of the middle point were added to the IOD process in addition to the observation set at the middle point.

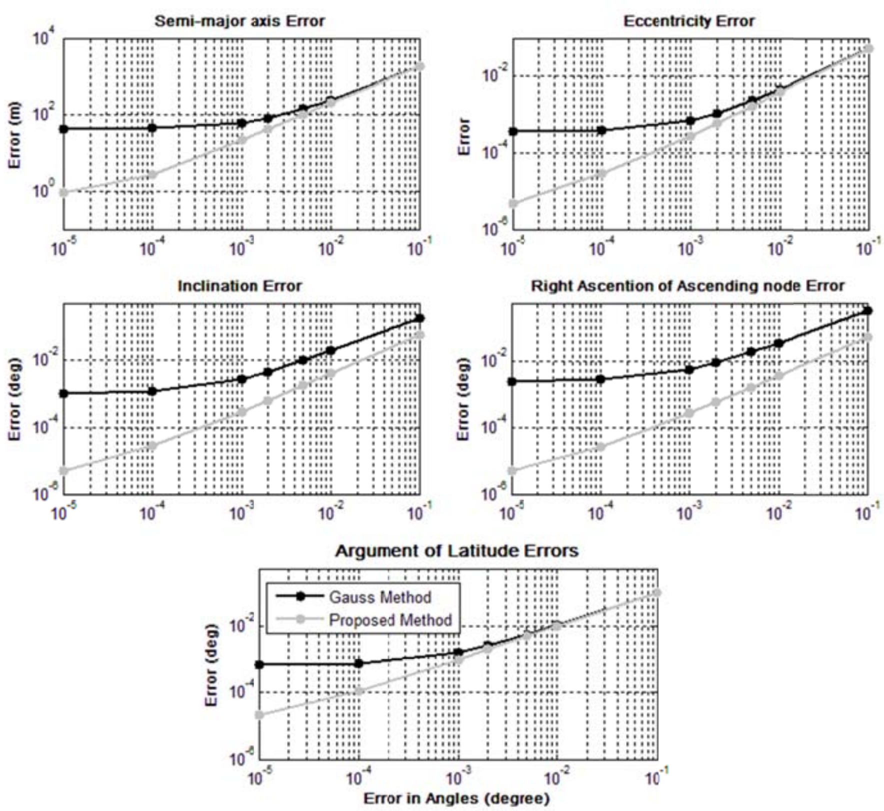

Figure 5. The error in Keplerian elements obtained from the Gauss and proposed methods on account of using the simulated noisy observations for a GPS satellite.

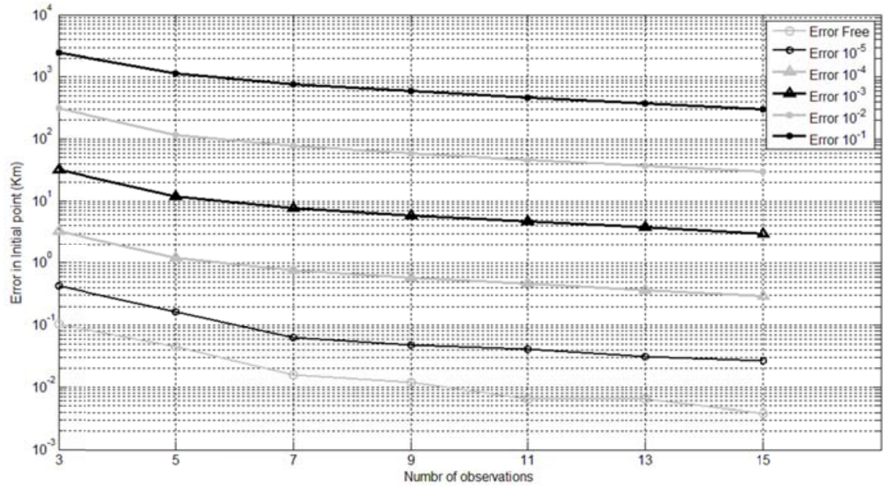

Figure 6. The IOD improvement using multi-observations for the GPS-02 satellite (simulated angles from real orbit in different levels of added Error of observations).

The details of Figure 6 have been represented in Table 1.

Table 1. The error at the initial point for the exact Gauss and proposed method for simulated observations with different level of noise (Case: GPS-02 satellite)

	Method	Num. of Observations	Level of error added to simulated angles				
			Error free	10^{-5}	10^{-4}	10^{-3}	10^{-2}
The Error in the middle point (Km)	Gauss Exact Method	3	7.774	8.0931	10.956	39.5021	316.5932
	Proposed Method	3	0.1044	0.4226	3.2858	31.8302	308.9092
		5	0.0443	0.1624	1.2252	11.8500	117.7815
		7	0.0158	0.0627	0.7693	7.8346	78.4101
		9	0.0121	0.0474	0.5824	5.9323	59.3899
		11	0.0065	0.0404	0.4625	4.6825	46.8532
		13	0.0064	0.0311	0.3690	3.7470	37.5055
		15	0.0038	0.0264	0.2984	3.0183	30.1998

As shown in Figure 6 and Table 1, the proposed method could improve the accuracy of IOD by about 70 times with respect to the exact Gauss in the error-free mode for a GPS-like satellite. Increasing accuracy could be continued by entering more observations. In the error-free mode, the 7.77 km of initial point error using the exact Gauss method is reduced to 3 m by using 15 observations. However, in more noised-polluted cases, there will not be any meaningful difference between the proposed and exact Gauss methods, because inaccurate observations do not need a more perfect model. In spite of this, entering more observations could obviously improve IOD even by highly noised observations.

Following the first scenario, the proposed method was validated by the real ground-track observations.

Under the second scenario, the real observations recorded in SLR stations were used for the validation of the proposed method. In the second scenario, the improvement of the Earth oblateness consideration and the process of adding more observations were tested by the real data. For this purpose, at first, the accurate angles (azimuth and elevation) were used for satellite position estimation. Following that, the estimated range obtained from IOD was compared with the accurate observed range. The algorithm of the procedure is shown in Figure 7.

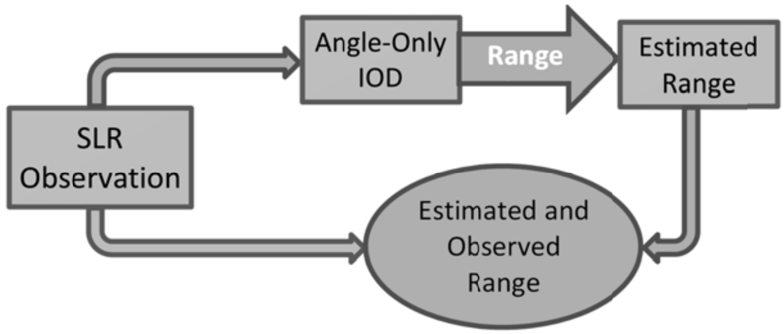

Figure 7. IOD Error Analysis using SLR Observations.

In the satellite laser ranging (SLR), a global network of the observation stations measures the round-trip time of the flight of the ultra-short pulses of light to satellites equipped with retro reflectors. This provides the instantaneous range measurements of the millimeter level precision, which can be accumulated to provide accurate measurement of orbits and a host of important scientific data.

The full-rate date of the SLR station, Matera in Italy, is used for validation of proposed method. The precision of the Azimuth and Elevation angles observed in the SLR station is about 0.1 milli-degree.

In Figure 7, the trend of improving the initial orbit determination is presented. In this figure, the error in the middle point is plotted versus the number of observations.

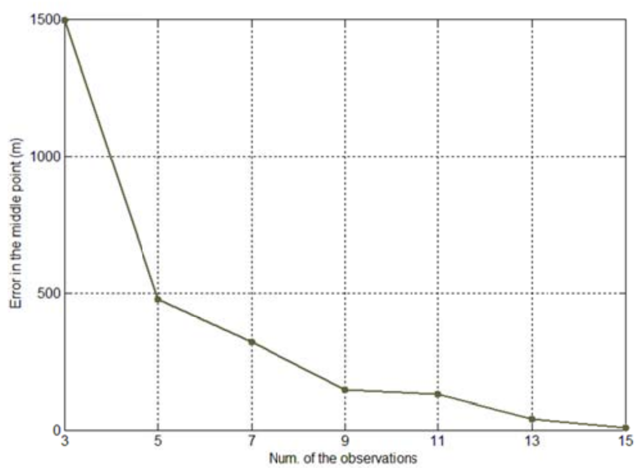

Figure 8. The IOD improvement using multi-observations for the GRACE A satellite (real observation recorded in Matera SLR station).

Table 2. The IOD improvement using the multi-observations for the GRACE A satellite.

Number of Observation sets	Error in the middle point (m)
3	1496.646
5	476.675
7	319.233
9	143.784
11	130.193
13	41.375
15	7.567

For more explanation, the error at the middle points (plotted in Figure 8) is listed in Table 2. For this test, the ground-based observations collected in the Matera station are used. As expected, the more the observations, the more accurate the initial orbit. The 1496.6 m of the initial point error is reduced to 7.5 m by increasing the number of the observations from 3 to 15.

4. Conclusions

The present study was designed to propose a new approach to the issue of the initial orbit determination based on the least square method. In addition to the Earth's oblateness consideration in the initial orbit determination as the most important perturbing acceleration, the proposed method is not limited in using more observations. In this paper, these findings suggest that considering the Earth's oblateness will be vital if the accurate observations are used, e.g. observations collected in the SLR stations. The Earth's oblateness consideration could improve the IOD accuracy six times for the LEO satellite (with short arc observations) and about sixty times for a MEO satellite (with long arc observations). Increasing the accuracy could be continued by using more observations. The accuracy of IOD was improved from 1496.6 m to 7.5 m by increasing the number of observations from 3 to 15, in the case of the ground-based observations of GRACE A satellite collected in the Matera station. It could be recommended that further research be undertaken with more focus on using other types of observation.

References

Bate, R. R., Mueller, D. D., Saylor, W. W., and White, J. E., 2013, Fundamentals of astrodynamics: (dover books on physics): Dover publications.

Celletti, A. and Pinzari, G., 2005, Four classical methods for determining planetary elliptic elements: a comparison. Celestial Mechanics and Dynamical Astronomy, 93(1-4), 1-52.

Celletti, A. and Pinzari, G., 2006, Dependence on the observational time intervals and domain of convergence of orbital determination methods Periodic, Quasi-Periodic and Chaotic Motions in Celestial Mechanics: Theory and Applications, pp. 327-344, Springer.

Cerri, L., Berthias, J., Bertiger, W., Haines, B., Lemoine, F., Mercier, F. and Ziebart, M., 2010, Precision orbit determination standards for the Jason series of altimeter missions. Marine Geodesy, 33(S1), 379-418.

Curtis, H. D., 2005, Orbital Mechanics for Engineering Students (Third edition. ed.).

Doscher, D. P., 2018, Orbit Determination for Space-Based Near Co-planar Observations of Space Debris Using Gooding's Method and Extended Kalman Filtering.

Escobal, P. R., 1965, Methods of Orbit Determination. New York: Wiley, 1965, 1.

Farnocchia, D., Tommei, G., Milani, A. and Rossi, A., 2010, Innovative methods of correlation and orbit determination for space debris. Celestial Mechanics and Dynamical Astronomy, 1071-2, 169-185.

Gauss, C. F., 2004, Theory of the Motion of the Heavenly Bodies Moving about the Sun in Conic Sections: Courier Dover Publications.

Gooding, R., 1996, A new procedure for the solution of the classical problem of minimal orbit determination from three lines of sight. Celestial Mechanics and Dynamical Astronomy, 6(64), 387-423.

Gronchi, G. F., 2009, Multiple solutions in preliminary orbit determination from three observations. Celestial Mechanics and Dynamical Astronomy, 103(4), 301-326.

Hu, J., Li, B. and Li, J., 2019, Initial orbit determination utilizing solution group optimization. IEEE Transactions on Aerospace and Electronic Systems.

Jäggi, A., Hugentobler, U., Bock, H. and Beutler, G., 2007, Precise orbit determination for GRACE using undifferenced or doubly differenced GPS data. Advances in Space Research, 39(10), 1612-1619.

Karimi, R. R. and Mortari, D., 2011, Initial orbit determination using multiple observations. Celestial Mechanics and Dynamical Astronomy, 109(2), 167-180.

Karimi, R. R. and Mortari, D., 2013, A performance based comparison of angle-only initial orbit determination methods.

Adv. Astronaut. Sci., AAS/AIAA, Hilton Head Island, South Carolina, 150, 1793-1809.

Kristensen, L. K., 2007, N-observations and radar orbits. Celestial Mechanics and Dynamical Astronomy, 98(3), 203-215.

Kristensen, L. K., 2009, Single lunation N-observation orbits. Celestial Mechanics and Dynamical Astronomy, 105(4), 275-287.

Laplace, P. S., 1780, Memoires de l' Académie royale des sciences de Paris. Collected Works, 10.

Lin, L. and Xin, W., 2003, A method of orbit computation taking into account the earth's oblateness. Chinese Astronomy and Astrophysics, 27(3), 335-339. doi: http://dx.doi.org/10.1016/S0275-1062(03)90056-7.

McCarthy, D. D. and Petit, G., 2003, IERS conventions. Paper presented at the IAU Joint Discussion.

McCutcheon, S. and McCutcheon, B., 2005, Space and astronomy: Infobase Publishing.

Merton, G., 1925, A modification of Gauss's method for the determination of orbits. Monthly Notices of the Royal Astronomical Society, 85, 693.

Milani, A. and Gronchi, G., 2010, Theory of orbit determination: Cambridge University Press.

Milani, A., Gronchi, G. F., Farnocchia, D., Knežević, Z., Jedicke, R., Denneau, L., and Pierfederici, F., 2008, Topocentric orbit determination: algorithms for the next generation surveys. Icarus, 195(1), 474-492.

Montenbruck, O. and Gill, E., 2000, Satellite orbits: Springer.

Sharifi, M. A. and Seif, M. R., 2011, Dynamic orbit propagation in a gravitational field of an inhomogeneous attractive body using the Lagrange coefficients. Advances in Space Research, 48(5), 904-913, doi: http://dx.doi.org/10.1016/j.asr.2011.04.021.

Taff, L. G., 1984, On initial orbit determination. The Astronomical Journal, 89, 1426-1428.

Tommei, G., Milani, A. and Rossi, A., 2007, Orbit determination of space debris: admissible regions. Celestial Mechanics and Dynamical Astronomy, 97(4), 289-304.

Vallado, D. A., 2001, Fundamentals of Astrodynamics and Applications (Vol. 12): Springer.

Van Helleputte, T. and Visser, P., 2008, GPS based orbit determination using accelerometer data. Aerospace Science and Technology, 12(6), 478-484.

Appendix

The Lagrange matrices have been formulated as:

$$\underline{F}(t_i)=\begin{bmatrix} f_1(t_i) & 0 & 0 \\ 0 & f_2(t_i) & 0 \\ 0 & 0 & f_3(t_i) \end{bmatrix}, \qquad \underline{G}(t_i)=\begin{bmatrix} g_1(t_i) & 0 & 0 \\ 0 & g_2(t_i) & 0 \\ 0 & 0 & g_3(t_i) \end{bmatrix} \tag{16}$$

The polynomial expressions for the coefficients were given using Taylor series expansion around the initial time t_0.

$$\begin{aligned} f_j(t_i) &= \sum_{n=0}^{\infty} \frac{1}{n!} f_j^{(n)}\Big|_{t=t_0} (t_i - t_n)^n \\ g_j(t_i) &= \sum_{n=0}^{\infty} \frac{1}{n!} g_j^{(n)}\Big|_{t=t_0} (t_i - t_n)^n \qquad j=1,2,3 \end{aligned} \tag{17}$$

Taylor expansion derived for a J2 field is presented in Table 3. As shown in Table 3, the coefficient $f_3^{(n)}$ and $g_3^{(n)}$ contain additional terms due to the non sphericity of the attracting force (Sharifi and Seif, 2011).

Table 3. Taylor expansion coefficients in J2 field.

n	$f_1^{(n)} = f_2^{(n)}$	$g_1^{(n)} = g_2^{(n)}$	$f_3^{(n)}$	$g_3^{(n)}$
0	1	0	1	0
1	0	1	0	1
2	$f_{\_kep}^{(2)} + a_{J_2}$	0	$f_1^{(2)} + e_{J_2}$	0
3	$f_{\_kep}^{(3)} + \dot{a}_{J_2}$	$g_{\_kep}^{(2)} + a_{J_2}$	$f_1^{(3)} + \dot{e}_{J_2}$	$g_1^{(3)} + e_{J_2}$
4	$f_{\_kep}^{(4)}$ $+\ddot{a}_{J2} - 2m\ a_{J2} + a_{J2}^2$	$g_{\_kep}^{(4)} + 2\dot{a}_{J2}$	$f_1^{(4)} + \ddot{e}_{J2} - 2m\ e_{J2} + e_{J2}^2$	$g_1^{(4)} + 2\dot{e}_{J_2}$
5	$f_{\_kep}^{(5)}$ $+a_{J2}^{(3)} + 3m\sigma\ a_{J2}$ $+\dot{a}_{J2}(-3m + 4a_{J2})$	$g_{\_kep}^{(5)}$ $+3\ddot{a}_{J2} + m\ a_{J2} + a_{J2}^2$	$f_1^{(5)} + e_{J_2}^{(3)} + 4\dot{e}_{J_2}(-m + a_{J_2})$ $+4e_{J_2}(3m\sigma + \dot{a}_{J_2})$	$g_1^{(5)} + 3\ddot{e}_{J_2}$ $+2e_{J_2}(-m + a_{J_2})$

In Table 4, $f_{\_kep}^{(i)}$ and $g_{\_kep}^{(i)}$ are the terms of the Lagrange coefficients in central field.

Table 4. Lagrange coefficients in central field.

n	$f_{\_kep}^{(n)}$	$g_{\_kep}^{(n)}$
0	1	0
1	0	1
2	$-m$	0
3	$3m\sigma$	$-m$
4	$-15m\sigma^2 + 3m\varepsilon + m^2$	$6m\sigma$
5	$105m\sigma^3 - \sigma(45m\varepsilon + 15m^2)$	$-45m\sigma^2 + 9m\varepsilon + m^2$

Appendix

The scalars a_{J_2}, e_{J_2} and their derivatives are defined as:

$$a_{J_2} = m'(5I_1^2 - 1)$$
$$\dot{a}_{J_2} = m'[5\sigma(1-7I_1^2) + 10I_1 I_2]$$
$$\ddot{a}_{J_2} = -m'[11mI_1^2 + m - 5\varepsilon(7I_1^2 - 1) + 10I_2^2 + 35m'\sigma^2(9I_1^2 - 1) - 140m'\sigma I_1 I_2]$$
$$a_{J_2}^{(3)} = m'[33m\sigma I_1^2 + (22mI_1 - 70\varepsilon I_1 + 630m'\sigma^2 I_1 - 140m'\sigma)(I_2 - I_1\sigma) + 5\sigma(m+2\varepsilon)(7I_1^2 - 1)$$
$$(\varepsilon - 2\sigma^2)(-140I_1 I_2 + 70\sigma(9I_1^2 - 1) - 175m'\sigma(4I_1 I_2 + 9I_1^2 - 1)$$
$$+20I_1(I_2 - 7m'\sigma)((-m + 5m'I_1 - 3m') - I_2\sigma)] \tag{18}$$

$$e_{J_2} = -2m'$$
$$\dot{e}_{J_2} = 10m'\sigma$$
$$\ddot{e}_{J_2} = -70m'\sigma^2 + 10m'\varepsilon \tag{19}$$
$$e_{J_2}^{(3)} = 50m'\sigma\left(7(\varepsilon - 2\sigma^2) + \sigma(m+2\varepsilon)\right)$$

Where

$$m' = (\frac{3J_2}{2})GM\,\frac{R^2}{r^5},$$
$$I_1 = \frac{z}{r}, \tag{20}$$
$$I_2 = \frac{\dot{z}}{r}.$$

where R is the Earth's radius and, J_2 is the second zonal harmonic coefficients.