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Average depth estimation of 2-D bodies through micro-gravity data by quasi Newton method

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Abstract

The function defined in MATLAB for minimizing the unconstrained multivariable function (fminunc) based on the quasi Newton method is used to minimize the square differences between calculated and observed data (misfit function).

The computer code provided is applied for estimating the average depth of n-sided polygons as synthetic models. The method is also used for real data.

Keywords: Average depth, Gravity data, Quasi Newton

1 INTRODUCTION

Depth is one of the most valuable parameters that can be determined through geophysical methods. Several methods are available to determine or estimate this parameter such as using the gravity derivatives, rule of thumb, least-square and Euler de convolution methods. Some of these methods are mostly applicable in the case of regular shape of the anomalies and some of them are quite subject to interpretation (Thompson, 1982).

In recent years by introducing efficient software like MATLAB, the numerical procedures have become more feasible than before.

The need for simple and straightforward methods for estimation of the depth of the bodies is quite sensible. We aim to provide a computer code in MATLAB to estimate the depth of the sources based on the minimization of the misfit function.

2 2-D n-SIDED POLYGON

The gravity attraction of an n-sided polygon in an arbitrary point can be calculated by Talwani's method (Grant and West, 1965),

$$g_{cal} = 2G\rho \sum_{k=1}^{nsides} \left\{ \frac{b_k}{1+a_k^2} \right\} \left\{ \frac{1}{2} \ln\left(\frac{x_{k+1}^2 + z_{k+1}^2}{x_k^2 + z_k^2} \right) + a_k \left\{ \tan^{-1}\left(\frac{x_{k+1}}{z_{k+1}} \right) - \tan^{-1}\left(\frac{x_k}{z_k} \right) \right\} \right\} \quad (1)$$

and,

$$a_k = \frac{x_{k+1} - x_k}{z_{k+1} - z_k}$$

$$b_k = \frac{x_k z_{k+1} - x_{k+1} z_k}{z_{k+1} - z_k} \quad (2)$$

where G is gravitational constant, ρ is contrast density between anomaly and host rocks, x and z are coordinates of the sides of the polygon.

3 MISFIT FUNCTION

The squared differences between gravity measurements (g_{obs}) and the gravity attraction of the polygon (g_{cal}) are defined as misfit function (Abdelrahman et al. 2001),

$$\varphi = \sum_{k=1}^{ndat} (g_{obs} - g_{cal})^2 \quad (3)$$

where ndat is the number of the measurement points. The aim is to calculate the unknown parameters (x and z of the sides of the polygon) in such a way that φ is minimum.

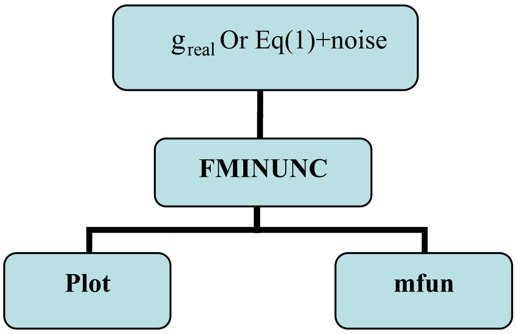
4 PROCEDURE

A computer code is provided in MATLAB. The input to the code are Bouguer gravity anomalies as observation data and the contrast density which has to be determined through prior information.

The observation data (g_{obs}) in the case of the models are prepared by equation (1) and contaminating them with random noises which are the most probable noises caused in data processing stage (Abdelrahman et al, 2001).

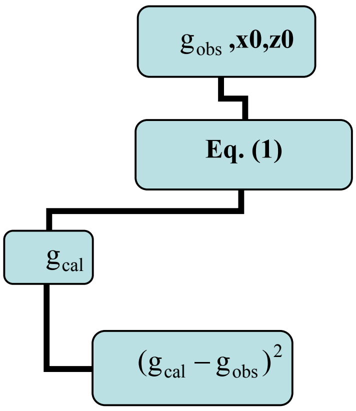
The random noises are generated by MATLAB function (NORMRND) as follows:
R=normrnd(MU,SIGMA,m,n), where MU and SIGMA are mean and standard deviation of normal random numbers (R) and m,n are the row of column dimension of R.

The calculated data (g_{cal}) are also computed by equation (1). For minimizing the misfit function (φ) the MATLAB function "Fminunc" is used. This function finds the minimum of a function of several variables.
X=Fminunc (FUN,X0) starts at X0 and finds a minimum X of the function FUN.
FUN accepts input X and returns a scalar function value F evaluated at X. X0 can be a scalar, vector or matrix. This function uses the quasi Newton method for optimization (Broyden, 1970). The procedure is reflected in Flowcharts (1) and (2).



Flowchart 1. Main Program.

g_{real} in this diagram means Bouguer gravity anomalies computed through measured values.



Flowchart 2. mfun.

5 SYNTHETIC MODELS

Several n-sided polygonal cross sections are used to show the effectiveness of the method. The models are demonstrated in graphs (1)-(4). The contrast density of the models are assumed to be 1. gr.cm^{-3} The depths of the models are shown in Table 1.

Table 1. Models, their minimum and maximum depths and computed average depth.

Models	Min. Depth (m)	Max. Depth (m)	Ave. Depth (m)
1	25	35	30
2	15	25	20
3	15	35	25
4	15	30	22.5

The results of the code are shown in Table 2.

Table 2. Models, their minimum and maximum depths and computed average depth.

Models	Max. noise (μGal)	No. Iterations	Ave. Depth(m)
1	2.5	70	25.95
2	3.39	84	23.5
3	1.5	85	19.5
4	1.4	89	20

Where the ave. depth is the average of the depths obtained after several iterations.

6 FIELD EXAMPLES

A micro-gravity survey has been carried out to detect the cavities and sink holes inside the power plant close to city of Hamadan in the west of Iran. The area is about 1km by 2 km.

The measurement stations are concerned mostly in the part of the area which contains man made facilities such as underground canals, cooling towers, boilers, gas oil reservoirs, etc.

Two gravity stations grids consisted of 1552 and 957 measurement points.
A basic grid dimension of 10 meters is used. The measurement point locations are also shown. Data were collected with a CG3-m gravity meter with a sensitivity of approximately 1μGal. In this area the thick Quaternary deposits are located on the lime-stone base rock which has been dissolved by underground water and caused cavities. These cavities have caused the sink holes in the area.

Two of the anomalies (low-density locations) detected are shown in figure 5 and figure 6 and the results of the code along profiles AB are shown in Table 3.

Table 3. Field models and their average depths.

Models	Max. noise (μGal)	No. Iterations	Ave. Depth (m)
Figure 5	-	54	4.5
Figure 6	-	65	23

The depth of these two anomalies are also detected through the Euler deconvolution method (Thompson, 1982) using Geosoft (version 5.1.5) and shown in figures 7 and 8.

The depth computed through the Euler method Figure 7 is very close to the depths obtained by our method along the profile in figure 5. In Figure 8 the depths of the north-earth part (green circles) are quite consistent with the depths along the profile in figure 6.

7 CONCLUSIONS

The method is quite capable of detecting the average depth of the anomalies.

However the computed depth is only the average and the top or the bottom depths should be computed through other methods.

ACKNOWLEDGEMENTS

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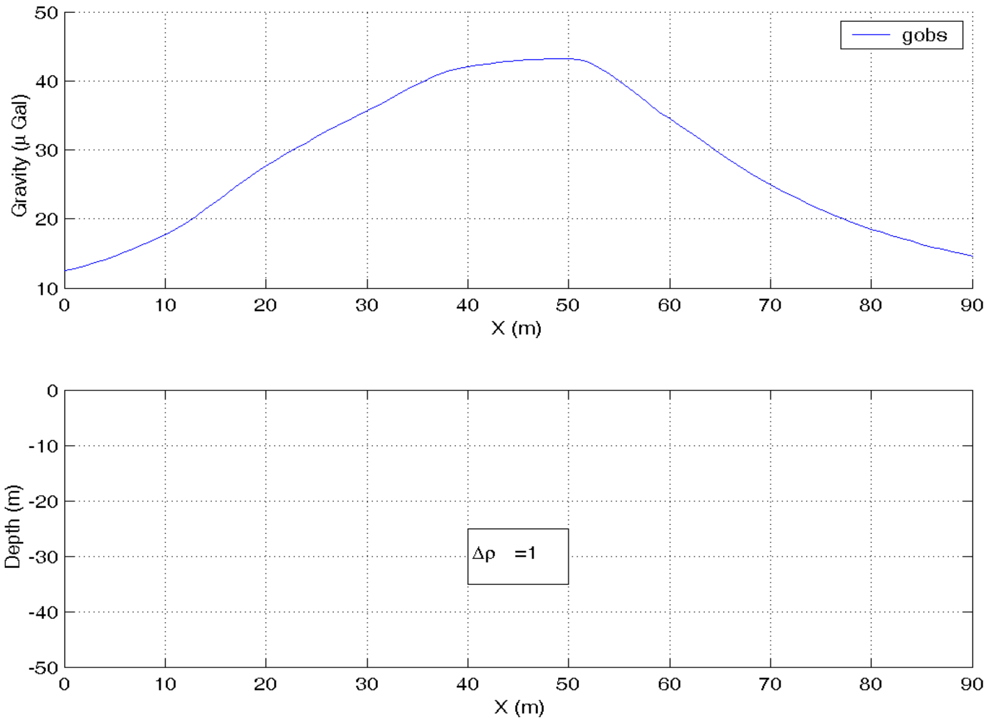
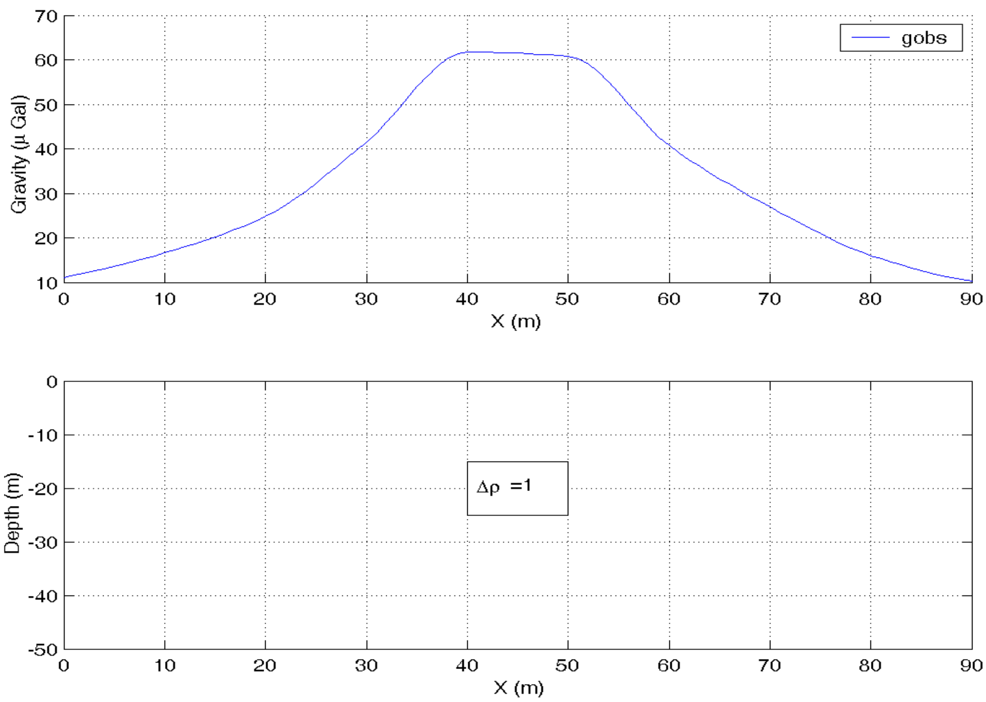
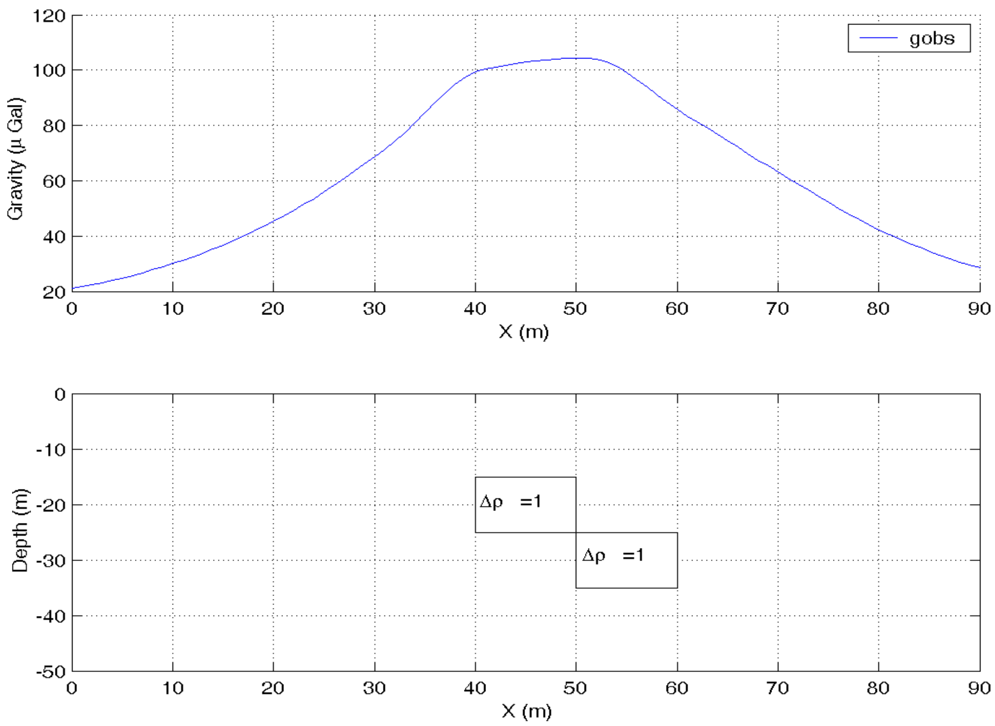


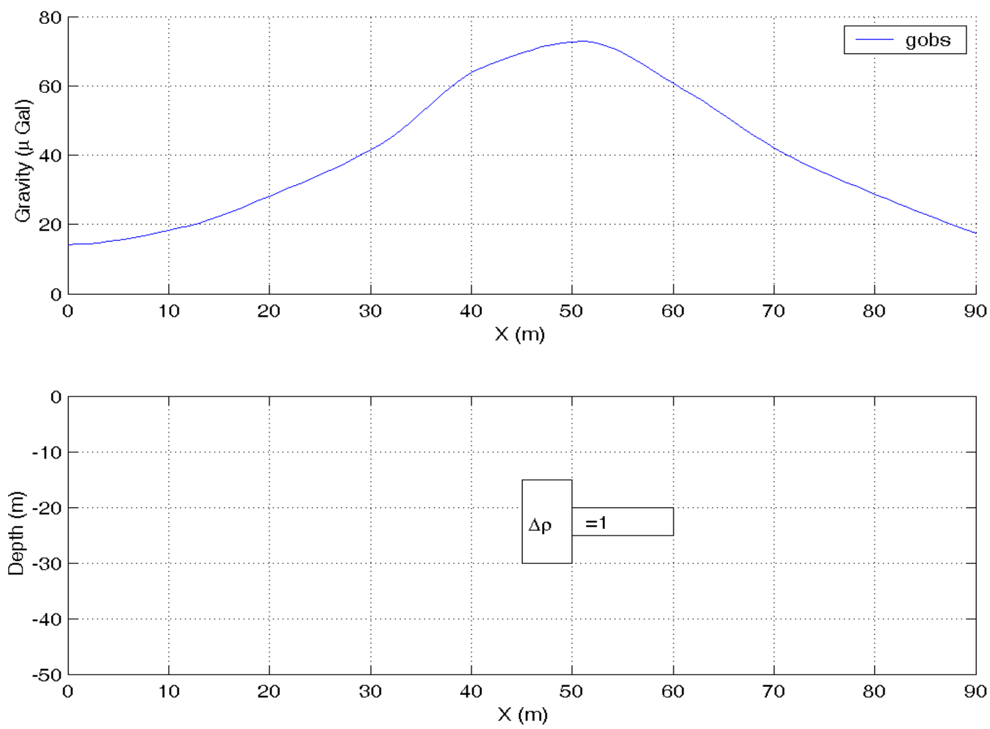
Graph. 1. Two-dimensional 4-sided polygonal cross section and its gravity effect contaminated by random noise.



Graph 2. Two-dimensional 4-sided polygonal cross section and its gravity effect contaminated by random noise.



Graph 3. Two-dimensional 4-sided polygonal cross sections and total gravity effect contaminated by random noise.



Graph 4. Two-dimensional 4-sided polygonal cross sections and total gravity effect contaminated by random noise.

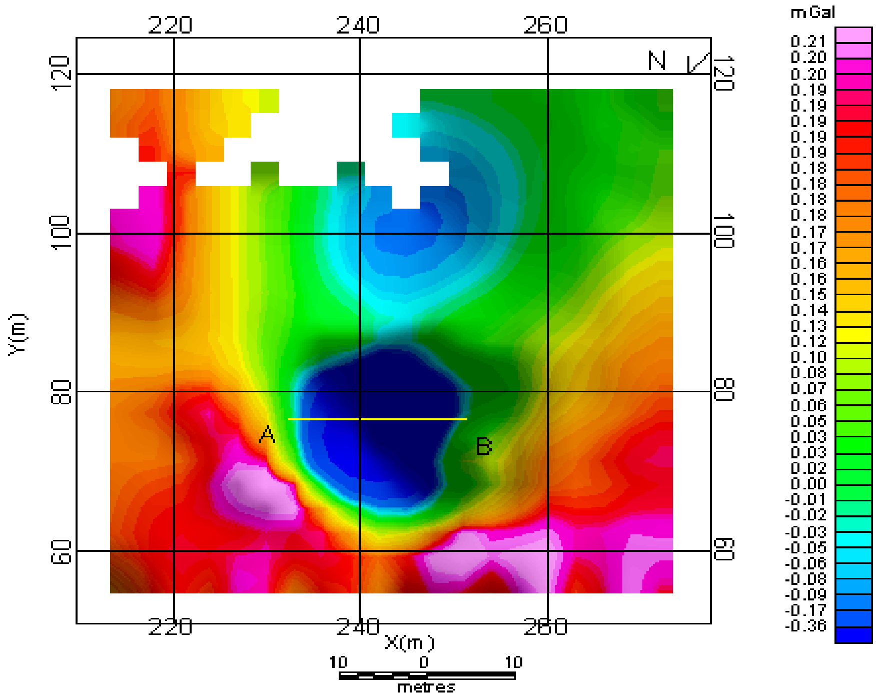


Figure 5. Bouguer gravity anomalies (mGal).

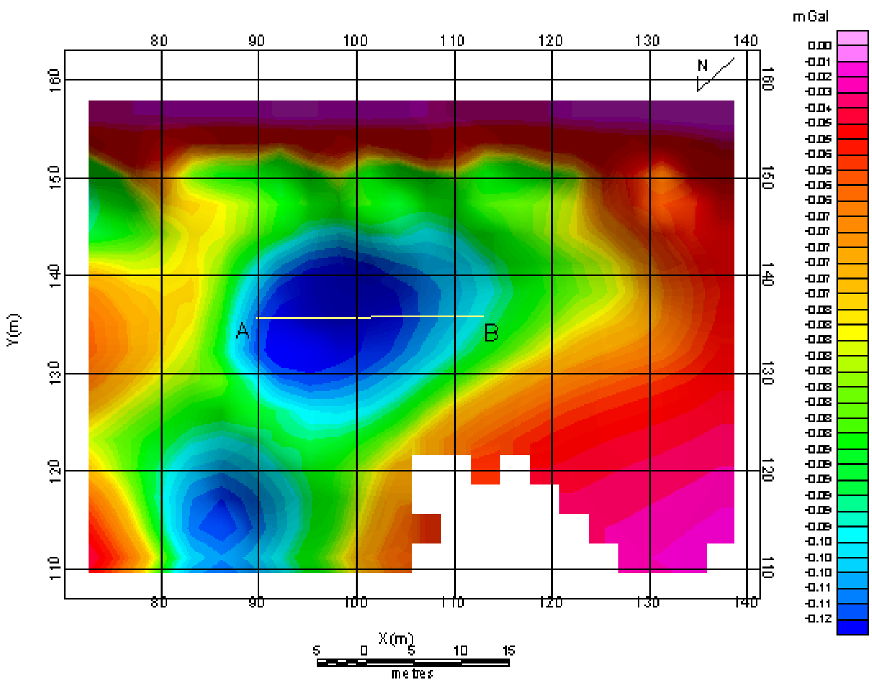


Figure 6. Bouguer gravity anomalies (mGal).

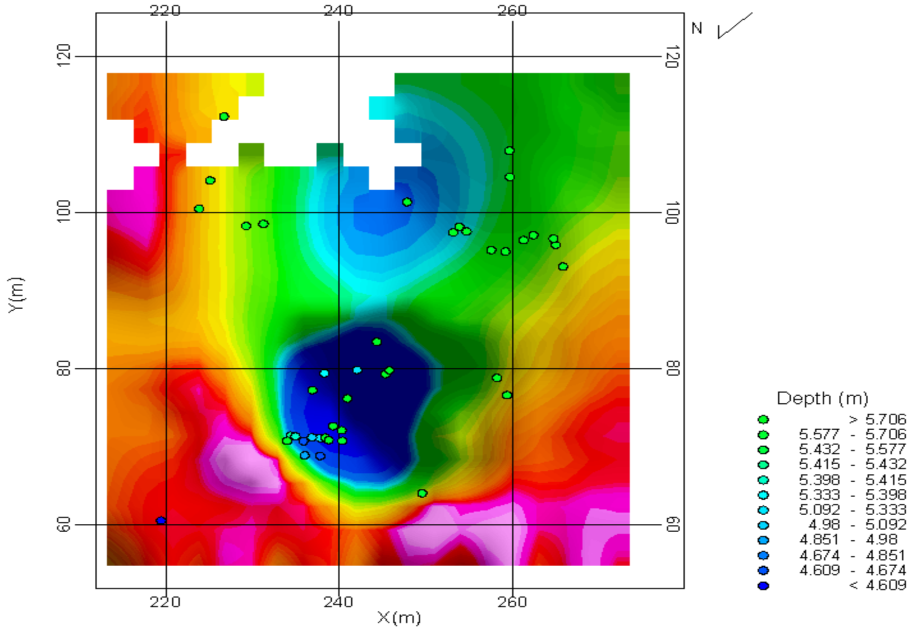


Figure 7. Euler depths (m).

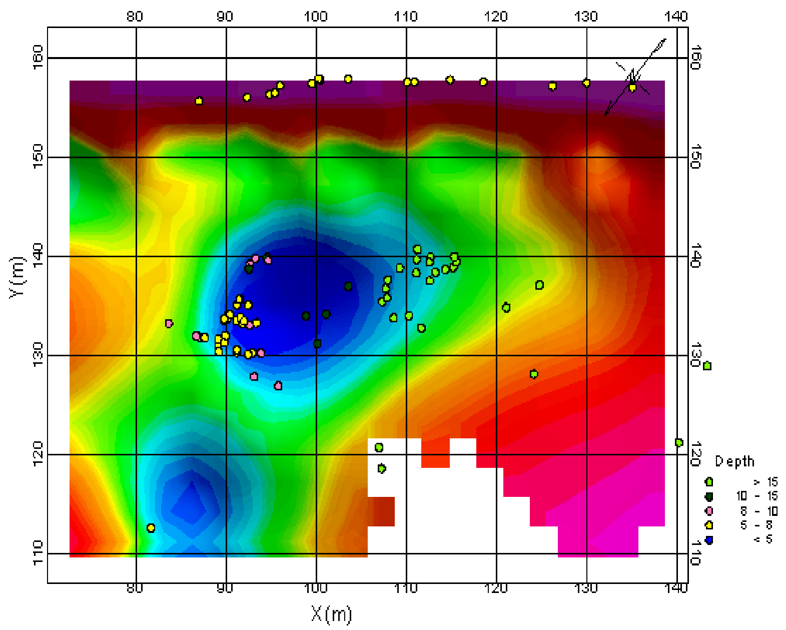


Figure 8. Euler depths (m).