Journal of the Earth & Space Physics. Vol. 32, No. 3, 2006, P. 1-9

The effect of solid tide in geopotential field of an elastic and inelastic earth

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(Received: 19 Aug 2003 , Accepted: 19 Sep 2006)

Abstract

In this paper the influence of solid tide on the Earth gravity field is considered. In this consideration the Earth can be regarded as either an elastic or inelastic body. Each one of these elastic and inelastic bodies has two main tidal components, frequency dependent and frequency independent components. In this article how to compute the effect of these components in the Earth's gravity field is presented. In this investigation, an attempt is made to find out whether the Earth should be regarded as an elastic or inelastic body in practical applications. Computations show equivalent effects on the gravity field due to the elastic and inelastic Earth model. The effect of the frequency dependent component of solid tide due to the elastic and inelastic Earth is much smaller than the frequency independent components. It depends on time and tidal constituents and it should be considered in precise applications. Comparisons between the solid tides due to the elastic and inelastic Earth model show that the inelastic Earth is contracted at poles about 3 mm and expanded at equator about 2.5 mm more than the elastic case.

Keywords: Tide, Elastic, Inelastic, Earth, Moon, Sun

1 INTRODUCTION

The changes induced by the solid Earth tides in the free space potential are most conveniently modeled as variations in the standard geopotential coefficients C_{nm} and S_{nm} (Eanes et al., 1983). The contributions ΔC_{nm} and ΔS_{nm} from the tides are expressible in terms of the k Love number using a simple spherical harmonic expansion equation (1). The effects of ellipticity and rotation of the Earth on tidal deformation necessities the use, in general, of three k parameters, k_{nm} and $k_{nm}^{(\pm)}$ to characterize the changes produced in free space potential by tides of spherical harmonic coefficients in degree (n) and order (m) (Wahr, 1981). Within the diurnal tidal band, for m=2 and n=1, these parameters have a strong frequency dependence due to the Nearly Diurnal Free Wobble resonance. Inelasticity of the mantle cause k_{nm} and $k_{nm}^{(\pm)}$ to acquire small imaginary parts reflecting a phase lag in deformation response of the Earth to tidal forces, and also gives rise to a further variation with frequency, which is particularly pronounced within the long period band. Trough modeling of inelasticity at the periods relevant to tidal phenomena (8 hours to 18.6 years) is not yet definitive, it is clear that the magnitudes of the contributions from inelasticity cannot be ignored, consequently the inelastic Earth model is recommended for precise applications.

The solid tide effect on the geopotential harmonic coefficient can easily be computed using the relations presented in annual IERS technical notes. In satellite orbit computations the solid tide plays a major role for low earth orbiting satellites and it is not negligible. Su (2000) did not consider the solid tide effect of inelastic earth for his orbit computations, because he seems to believe that the effect of higher degree and order of solid tide variations in geopoential coefficients are negligible for GEO (Geostationary) and MEO (Medium Earth Orbiting) satellites. Buffet (1985) used only k_{20} coefficient for his computations. Wolf (2000) also employed the higher degree and order of nominal Love number for LEO (Low Earth Orbiting) satellites, but he just considered the elastic earth. Also Eanes (1983) suggested using an elastic Earth for considering the solid tide. Finally, Eshagh (2003) shows that there is no significant difference between two cases where the earth is consider either elastic or inelastic for orbit computations even for LEO satellites. He believes that the high altitude of the satellite diminishes this difference. But in this paper, we are going to reduce the altitude to a surface close to the Earth's surface and show the difference between elastic and inelastic solid tide effects. In other words the uplifts due to the solid tide in an elastic and inelastic earth are investigated.

In the following sections of this paper, the effects of the solid tide on the geopotentioal harmonic coefficients for an elastic and inelastic Earth model and their mathematical expressions are presented and some numerical studies are done too.

2 EFFECT OF SOLID TIDE ON GEOPOTENTIAL HARMONIC COEFFICIENTS

The computation of the tidal contributions to the geopotential coefficients is mostly done in two steps. In the first step the (2m) part of the tidal potential of coefficients is evaluated in the time domain for each m by using lunar and solar ephemeris, and the corresponding changes ΔC_{2m} and ΔS_{2m} are computed using frequency independent nominal values k_{2m}. The contribution of the degree 3 tides to C_{3m} and S_{3m} through k_{3m} and also those of the degree 2 tides to C_{4m} and S_{4m} through $k_{2m}^{(+)}$ may be computed by a similar procedure. In the second step the corrections for deviations of k_{21} of several of the constituent tides of the diurnal band from the constant nominal value k_{21} are assumed for this band in the first step. Similar corrections need to be applied to a few of the constituents of the other two bands also (McCarthy, 1996). In the following section step 1 and step 2 of tidal contributions to the geopotential coefficients are mathematically presented.

3 FREQUENCY INDEPENDENT EFFECTS IN AN ELASTIC EARTH

According to the above description, the frequency independent values k_{nm}, changes induced by the (nm) part of the tidal generating potential in the normalized geopotential coefficients having the same (nm) are given in the time domain by (McCarthy, 1996).

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1} \bar{P}_{nm}\left(\sin\Phi_j\right)e^{-im\lambda_j} \qquad (1)$$

where, k_{nm} is the nominal degree Love number for degree n and order m, R_e equatorial radius of the Earth, $GM_\oplus$ gravitational parameter for the Earth, GM_j gravitational parameter for the Moon (j=2) and Sun (j=3), r_j distance from geocenter of the Moon or Sun, Φ_j body fixed geocentric latitude of the Moon or Sun, λ_i body fixed east longitude of Moon or Sun, $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$ are the contributions of the tidal potential to the normalized geopotential harmonic coefficients, and $\bar{P}_{nm}$ is the normalized associated Legendre function, (Santos, 1994).

$$\bar{P}_{nm} = N_{nm}P_{nm}, \qquad (2)$$

Where

$$N_{nm} = \sqrt{\frac{(n-m)!(2n+1)(2-\delta_{0m})}{(n+m)!}} \qquad (3)$$

where, δ_{0m} is the Kroneker delta. As one can see the contributions of the tidal potential can form corrections to the geopotential harmonic coefficients up to degree and order 3. The influence of higher degree and order are so small and can be neglected and they are at the level of 10^{-11}. Further computations can be done if one wants to consider higher degree and order. The effect on the geopotential coefficients in degree of 4 can be expresses in terms of the $k_{2m}^{(+)}$ as follows (McCarthy, 1996, McCarthy and Petit, 2003).

$$\Delta\bar{C}_{4m} - i\Delta\bar{S}_{4m} = \frac{k_{2m}^{(+)}}{5}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1} \bar{P}_{nm}\left(\sin\Phi_j\right)e^{-im\lambda_j} \quad (m=0,1,2) \qquad (4)$$

which has the same form as the previous equation for n=2 except for the replacement of k_{2m} by $k_{2m}^{(+)}$. The values of nominal Love numbers are given in the following table. The choice of these values have been made so as to minimize the number of terms for which corrections will have to be applied in the next step. The nominal value for m=0 has to be chosen as real because the contribution to $\bar{C}_{20}$ from the imaginary part of $k_{20}^{(0)}$.

Table 1. Nominal frequency independent values of solid tide external potential Love number.

Degree	**Order**	**Elastic Earth**		**Inelastic Earth**		
n	m	k_{nm}	$k_{nm}^{(+)}$	$\mathrm{Re}\,k_{nm}$	$\mathrm{Im}\,k_{nm}$	$k_{nm}^{(+)}$
2	0	0.29525	-0.00087	0.30190	-0.00000	-0.00089
2	1	0.29470	-0.00079	0.29830	-0.00144	-0.00080
2	2	0.29801	-0.00057	0.30102	-0.00130	-0.00057

4 FREQUENCY INDEPENDENT EFFECTS IN AN INELASTIC EARTH

For an inelastic Earth the nominal values of Love number must be used via complex numbers (McCarthy, 1996). By substituting the complex Love numbers in equation (1) we have.

$$\Delta\overline{C}_{nm} - i\Delta\overline{S}_{nm} = \frac{(\mathrm{Re}\,k_{nm} + i\,\mathrm{Im}\,k_{nm})}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)e^{-im\lambda_j} \quad (5)$$

$$\Delta\overline{C}_{nm} - i\Delta\overline{S}_{nm} = \frac{(\mathrm{Re}\,k_{nm} + i\,\mathrm{Im}\,k_{nm})}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)(\cos m\lambda - i\sin m\lambda) \quad (6)$$

$$\Delta\overline{C}_{nm} - i\Delta\overline{S}_{nm} = \frac{1}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)((\cos m\lambda\,\mathrm{Re}\,k_{nm} + \sin m\lambda\,\mathrm{Im}\,k_{nm}) + i(\cos m\lambda\,\mathrm{Im}\,k_{nm} - \sin m\lambda\,\mathrm{Re}\,k_{nm}))$$

$$\Delta\overline{C}_{nm} - i\Delta\overline{S}_{nm} = \frac{1}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)(\mathrm{Re}\,k_{nm} + i\,\mathrm{Im}\,k_{nm})(\cos m\lambda - i\sin m\lambda) \quad (8)$$

All of the parameters have been defined previously except $\mathrm{Re}\,k_{nm}$ and $\mathrm{Im}\,k_{nm}$, which are the real and imaginary part of the complex Love numbers respectively.

5 FREQUENCY DEPENDENT EFFECTS IN AN ELASTIC EARTH

The frequency dependent values for use in step 2 are taken from the results of computations by Mathews and Buffett using PREM elastic Earth model with the ocean layer replaced by solid, and for evaluation of inelasticity effects, the Widmer et al 1991 model of mantle Q. As in Wahr and Bergen (1986), a power law was assumed for the frequency dependence of Q with 200s as the reference period; the inelasticity contribution (out of phase and in-phase) to the tidal changes in the geopotential coefficients is at the level of one to two percent in-phase and half to one percent out of phase and of the order of 10^{-10} (McCarthy, 1996). The frequency dependence corrections to geopotential coefficients obtained from the first step are also computed in second step as the sum of contributions from a number of tidal constituents belonging to the respective bands. The contribution to the second zonal harmonic coefficients from the long period tidal constituents of various frequencies is (Mc Carthy, 1996, McCarthy and Petit, 2003).

$$\mathrm{Re}\sum_{f(2,0)}(A_0\delta k_f H_f e^{i\theta_f}) = \sum_{f(2,0)}(A_0 H_f(\delta k_f^R\cos\theta_f - \delta k_f^i\sin\theta_f)) \quad (9)$$

The contribution due to the diurnal tidal constituents and semidiurnal constituents are given as follow (McCarthy, 1996, McCarthy and Petit, 2003).

$$\Delta\overline{C}_{2m} - \Delta\overline{S}_{2m} = \eta_m\sum_{f(2,m)}(A_m\delta k_f H_f)e^{i\theta_f} \qquad (m = 1, 2) \quad (10)$$

where,

$$A_0 = \frac{1}{R_e\sqrt{4\pi}} = 4.4228\times10^{-8}\,m^{-1}, \quad (11)$$

$$A_m = \frac{(-1)^m}{R_e\sqrt{8\pi}} = (-1)^m(3.1274\times10^{-8})m^{-1}$$

$$m \neq 0, \quad \eta_1 = -1, \quad \eta_2 = 2, \quad (12)$$

where, δk_f, δk_f^R, δk_f^I, H_f are differences between k_{2m} at frequency f and nominal value k_{2m}, and real part of δk_f, imaginary part of δk_f, amplitude of the term at frequency f from the harmonic expansion of the tidal potential respectively, θ_f is defined as follow.

$$\theta_f = m(\theta_g + \pi) - \sum_{j=1}^{5} N_j F_j, \qquad (13)$$

where, F_j fundamental arguments (Delaunay variables l, l', F, D, Ω) of nutation theory, N_j multiplier of Delaunay variables for nutation and θ_g is the Greenwich Mean Sidereal Time expressed in angle unit. The fundamental arguments are L the mean anomaly of the Moon, L' the mean anomaly of the apparent Sun, Ω the longitude of the ascending node of the moon's orbit, F the mean latitude of the Moon, and D the mean elongation of longitude of the Moon and Sun. The expression of these fundamental arguments (McCarthy, 1996, McCarthy and Petit, 2003, Eshagh, 2003),

$$F_1 = L = 134^\circ.96340251 + 1717915923''.2178T_d + 31''.8792T_d^2 + 0''.051635T_d^3 - 0''.00024470T_d^4, \qquad (14)$$

$$F_2 = L' = 357^\circ.52910918 + 129596581''.0481T_d - 0''.5532T_d^2 + 0''.000136T_d^3 - 0''.00001149T_d^4, \qquad (15)$$

$$F_3 = \Omega = 125^\circ.04455501 - 6962890''.2665T_d + 7''.4722T_d^2 + 0''.007702T_d^3 - 0''.00005939T_d^4, \qquad (16)$$

$$F_4 = F = 93^\circ.27209062 + 1739527262''.8478T_d - 12''.7512T_d^2 - 0''.001037T_d^3 - 0''.00000417T_d^4, \qquad (17)$$

$$F_5 = D = 297^\circ.85019547 + 1602961601''.2090T_d - 6''.3706T_d^2 + 0''.006593T_d^3 - 0''.00003169T_d^4, \qquad (18)$$

where

$$T_d = \frac{TT - 2451545.0}{36525}, \qquad (19)$$

and, TT is the terrestrial time in the Julian date. The multipliers of Delaunay variables are accessible from IERS technical reports. The Greenwich Mean Sidereal Time is computed from following equation (Santos, 1994).

$$GMST = UT1\frac{1}{r'} + 0UT \qquad (20)$$

where, 0UT is the shift between sidereal time and mean time (Santos, 1994, Eshagh, 2003).

$$0UT = 24110''.54841 + 8640184''.812866T_d + 0''.093104T_d^2 - 6''.2 \times 10^{-6} T_d^3, \qquad (21)$$

and $\frac{1}{r'}$ is the scale factor between sidereal time and mean time (Santos, 1994, Eshagh, 200).

$$\frac{1}{r'} = 1.002737909350795 + 5.9006 * 10^{-11} T_d - 5.9 * 10^{-15} T_d^2, \qquad (22)$$

The amplitude of tidal constituents has been presented in IERS technical reports. After introducing these equations the θ_f is computed easily.

6 FREQUENCY DEPENDENT EFFECTS IN AN INELASTIC EARTH

There is no second step correction for second zonal harmonic coefficients (McCarthy, 1996). The corrections due to the semidiurnal band of tide are the same for elastic and inelastic Earth (McCarthy, 1996). The relations for frequency dependence effects for inelastic Earth are the same. For computing other effects there are some tables computed by IERS for inelastic Earth. The frequency dependent effects regarding the Earth as an inelastic body, depends on the tidal constituent. The influences on the geopotential coefficients can be computed by replacing δk_f in equation (9) by (McCarthy and Petit, 2003).

$$\delta k_f = \delta k_f^R + i\delta k_f^I, \qquad (23)$$

and the following relations can compute the corrections (McCarthy and Petit, 2003).

$$(\Delta \overline{C}_{2m})_{constituent} = A_m H_f (\delta k_f^R \sin\theta_f + \delta k_f^I \cos\theta_f), \qquad (24)$$

$$(\Delta \overline{S}_{2m})_{constituent} = A_m H_f (\delta k_f^R \cos\theta_f - \delta k_f^I \sin\theta_f. \qquad (25)$$

The following table lists the results for all tidal terms which contribute 10^{-13} or more, after rounding off to the (nm)=(21) geopotential coefficient. A cutoff at this level is used for the individual terms in order that accuracy at the level of 3×10^{-12} be not affected by the accumulated contributions from the numerous smaller terms that are disregarded. The imaginary parts of the contributions are below cutoff and are not listed.

In the above table one can see the name of some tidal constituents as well as their rates. Also the Doodson numbers and coefficients of the Doodson and fundamental Delaunay arguments are presented. The difference between reference nominal second Love number and the Love numbers, and their amplitudes, in the elastic and inelastic Earth model can also be seen.

Table 2. Results for all tidal terms.

Name	Deg/hr	Doodson No.	τ s h p N′ p_s	l l′ F D Ω	δk_f^{el}	Amp. Elas.	δk_f^{inel}	Amp. Inelas.
	13.39645	135.645	1–2 0 1–1 0	1 0 2 0 1	-0.00044	-0.1	-0.00045	-0.1
Q_1	13.39866	135.655	1–2 0 1 0 0	1 0 2 0 2	-0.00044	-0.7	-0.00046	-0.7
ρ_1	13.47151	137.455	1–2 2–1 0 0	-1 0 2 2 2	-0.00047	-0.1	-0.00049	-0.1
	13.94083	145.454	1–1 0 0–1 0	0 0 2 0 1	-0.00081	-1.2	-0.00082	-1.3
O_1	13.94303	145.555	1–1 0 0 0 0	0 0 2 0 2	-0.00081	-6.6	-0.00082	-6.7
$N\tau_1$	14.41456	153.655	1 0–2 1 0 0	1 0 2–2 2	-0.00167	0.1	-0.00168	0.1
LK_1	14.48741	155.455	1 0 0–1 0 0	-1 0 2 0 2	-0.00193	0.4	-0.00193	0.4
NO_1	14.49669	155.655	1 0 0 1 0 0	1 0 0 0 0	-0.00196	1.3	-0.00197	1.3
	14.49890	155.665	1 0 0 1 1 0	1 0 0 0 1	-0.00197	0.2	-0.00198	0.3
χ_1	14.56955	157.455	1 0 2–1 0 0	-1 0 0 2 0	-0.00231	0.3	-0.00231	0.3
π_1	14.91787	162.556	1 1–3 0 0 1	0 1 2–2 2	-0.00834	-1.9	-0.00832	-1.9
	14.95673	163.545	1 1–2 0–1 0	0 0 2–2 1	-0.01114	0.5	-0.01111	0.5
P_1	14.95893	163.555	1 1 –2 0 0 0	0 0 2–2 2	-0.01135	-43.3	-0.01132	-43.2
S_1	15.00000	164.556	1 1–1 0 0 1	0 1 0 0 0	-0.01650	2.0	-0.01642	2.0
	15.03886	165.545	1 1 0 0–1 0	0 0 0 0 -1	-0.03854	-8.8	-0.03846	-8.8
K_1	15.04107	165.555	1 1 0 0 0 0	0 0 0 0 0	-0.04093	472.0	-0.04085	471.0
	15.04328	165.565	1 1 0 0 1 0	0 0 0 0 1	-0.04365	68.3	-0.04357	68.2
	15.04548	165.575	1 1 0 0 2 0	0 0 0 0 2	-0.04678	-1.6	-0.04670	-1.6
ψ_1	15.08214	166.554	1 1 1 0 0–1	0–1 0 0 0	0.23083	-20.8	0.22609	-20.4
ϕ_1	15.12321	167.555	1 1 2 0 0 0	0 0–2 2–2	0.03051	-5.0	0.03027	-5.0
θ_1	15.51259	173.655	1 2–2 1 0 0	1 0 0–2 0	0.00374	-0.5	0.00371	-0.5
J_1	15.58545	175.455	1 2 0–1 0 0	-1 0 0 0 0	0.00329	-2.1	0.00325	-2.1
	15.58765	175.465	1 2 0–1 1 0	-1 0 0 0 1	0.00327	-0.4	0.00324	-0.4
SO_1	16.05697	183.555	1 3–2 0 0 0	0 0 0–2 0	0.00198	-0.2	0.00195	-0.2
OO_1	16.13911	185.555	1 3 0 0 0 0	0 0–2 0–2	0.00187	-0.7	0.00184	-0.6
	16.14131	185.565	1 3 0 0 1 0	0 0–2 0–1	0.00187	-0.4	0.00184	-0.4

The nominal value k_{20} for the zonal tides is taken as 0.30190. The real and imaginary parts δk_f^R and δk_f^I are listed, along with the corresponding in phase (ip) amplitude $(A_0 H_f \delta k_f^R)$ and out of phase (op) amplitude $(A_0 H_f \delta k_f^I)$ to be used in equation (9). In the elastic case, $k_{20} = 0.29$ for all zonal tides, and no second step corrections are needed.

Table 3. Corrections for frequency dependence of k_{20} of the zonal tides due to inelasticity, unit 10^{-12}.

Name	Deg/hr	Doodson No.	τ s h p N′ p_s	l l′ F D Ω	δk_f^R	Amp. (ip)	δk_f^I	Amp. (op)
	0.00221	55.565	0 0 0 0 1 0	0 0 0 0 1	0.01347	16.6	-0.00541	-6.7
	0.00441	55.575	0 0 0 0 2 0	0 0 0 0 2	0.01124	-0.1	-0.00488	0.1
S_a	0.04107	56.554	0 0 1 0 0–1	0–1 0 0 0	0.00547	-1.2	-0.00349	0.8
S_{sa}	0.08214	57.555	0 0 2 0 0 0	0 0–2 2–2	0.00403	-5.5	-0.00315	4.3
	0.08434	57.565	0 0 2 0 1 0	0 0–2 2–1	0.00398	0.1	-0.00313	-0.1
	0.12320	58.554	0 0 3 0 0–1	0–1–2 2–2	0.00326	-0.3	-0.00296	0.2
M_{sm}	0.47152	63.655	0 1–2 1 0 0	1 0 0–2 0	0.00101	-0.3	-0.00242	0.7
	0.54217	65.445	0 1 0–1–1 0	-1 0 0 0–1	0.00080	0.1	-0.00237	-0.2
M_m	0.54438	65.455	0 1 0–1 0 0	-1 0 0 0 0	0.00080	-1.2	-0.00237	3.7
	0.54658	65.465	0 1 0–1 1 0	-1 0 0 0 1	0.00079	0.1	-0.00237	-0.2
	0.55366	65.655	0 1 0 1 0 0	1 0–2 0–2	0.00077	0.1	-0.00236	-0.2
M_{sf}	1.01590	73.555	0 2–2 0 0 0	0 0 0–2 0	-0.00009	0.0	-0.00216	0.6
	1.08875	75.355	0 2 0–2 0 0	-2 0 0 0 0	-0.00018	0.0	-0.00213	0.3
M_f	1.09804	75.555	0 2 0 0 0 0	0 0–2 0–2	-0.00019	0.6	-0.00213	6.3
	1.10024	75.565	0 2 0 0 1 0	0 0–2 0–1	-0.00019	0.2	-0.00213	2.6
	1.10245	75.575	0 2 0 0 2 0	0 0–2 0 0	-0.00019	0.0	-0.00213	0.2
M_{stm}	1.56956	83.655	0 3–2 1 0 0	1 0–2–2–2	-0.00065	0.1	-0.00202	0.2
M_{tm}	1.64241	85.455	0 3 0–1 0 0	-1 0–2 0–2	-0.00071	0.4	-0.00201	1.1
	1.64462	85.465	0 3 0–1 1 0	-1 0–2 0–1	-0.00071	0.2	0.00201	0.5
M_{sqm}	2.11394	93.555	0 4–2 0 0 0	0 0–2–2–2	-0.00102	0.1	0.00193	0.2
M_{qm}	2.18679	95.355	0 4 0–2 0 0	-2 0 2 0–2	-0.00106	0.1	0.00192	0.1

Table 4. Amplitudes $(A_2 \delta k_f H_f)$ of corrections for frequency dependent of k_{22}.

Name	Deg/hr	Doodson No.	τ s h p N′ p_s	l l′ F D Ω	δk_f^R	Amp.
N_2	28.43973	245.655	2–1 0 1 0 0	1 0 2 0 2	0.00006	-0.3
M_2	28.98410	255.555	2 0 0 0 0 0	0 0 2 0 2	0.00004	-1.2

Taking the nominal value k_{22} for sectorial tides as 0.29801 for the elastic case and (0.30102-0.00130i) for the inelastic case. Units: 10^{-12} the corrections are only to the real part, and are the same in both elastic and inelastic cases.

The total variation in geopotential coefficient $\overline{C}_{20}$ is obtained by adding to the result of step 1 the sum of the contributions from the tidal constituents listed in table 3 computed using equation (9) The tidal variations in $\overline{C}_{2m}$ and $\overline{S}_{2m}$ for the other m are computed similarly except equation (10) is to be used together with table 2 for m=1 and table 4 for m=2.

7 NUMERICAL RESULTS

The effects of the solid tide can be expressed as uplift; the potential variations due to the solid tide can be expressed by using harmonic expansion of gravity field also this potential variation can be converted to the uplift as follow

$$\text{uplift} = \frac{\Delta U}{g}, \tag{26}$$

according to the above relation the uplifts can be computed using the normal gravity field of the Earth. The following figure shows the magnitude of the uplift considering the Earth as an elastic body.

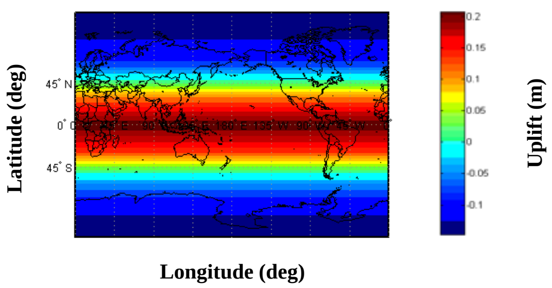

Figure 1. The uplifts due to the solid tide on an elastic Earth (frequency independence correction).

According to the figure it is concluded that most uplifts have occurred on the equator. The value of uplifts is reduced as the absolute value of latitude increased up to the latitude of about 45 and -45 degree. The small uplifts can be seen in the regions their latitudes are larger than 45 and smaller than -45 degrees. The uplift values are reduced from these regions to the poles. It shows that the ellipticity of the Earth is increased by the solid tide.

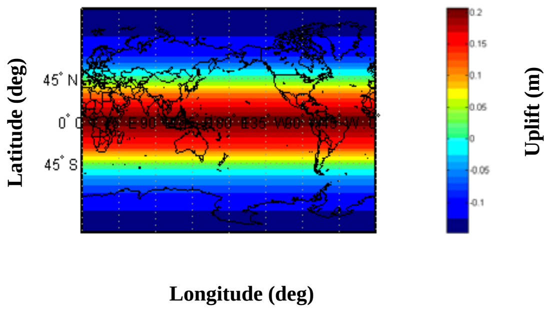

Figure 2. The uplifts due to the solid tide on an inelastic Earth (frequency dependence correction).

By considering the Earth as an inelastic body, one can see similar uplifts on the figure. Also similar expression can be stated for this case, namely maximum uplift occurs on the equator about 0.2 m and minimum uplift occurs on the poles. By considering these two figures one can say that there is no significant difference between both cases either the Earth is regarded as an elastic or inelastic body. For further investigations let us to consider the differences between the uplifts obtained from both cases. The following figure presents these differences on the Earth.

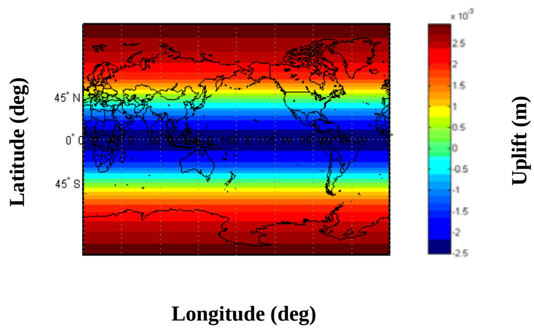

Figure 3. Difference between uplifts due to the solid Earth tide in an elastic and inelastic Earth (frequency independence corrections).

According to the figure, it is seen that maximum differences are around the poles, and the minimum uplift difference occurs on the equator, also one can see that the magnitude of the uplift differences are very small and about the a few mm. From a magnitude point of view, it is possible to say that in the regions with 45 and -45 degrees of latitude, the magnitude of the uplift differences is small. Finally it can be said that when the Earth is regarded as an inelastic body the solid tide increases the ellipticity of the Earth more than when the Earth is considered as an elastic body.

Now the effect of the frequency dependent components of the solid tide is considered. The uplifts due to this effect on an elastic Earth can be seen as follows

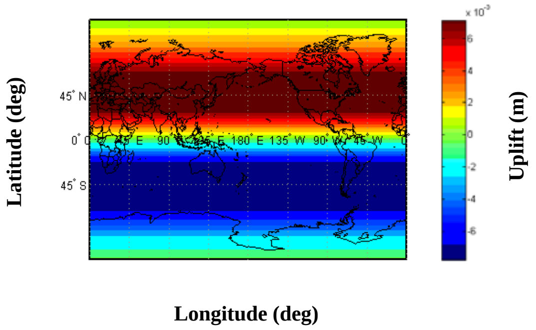

Figure 4. The uplifts due to the solid tide on an elastic Earth (frequency dependence correction).

As we can see, the maximum uplift happened in the regions with latitudes of about 45 degrees. This effect does not depend on the position of the Moon and the Sun but it is related to the time and tidal constituents directly. The minimum uplift can be seen in the latitude of -45 degrees. Also the figures show that the magnitude of the uplifts is small with respect to the frequency independent cases. If an inelastic Earth is considered we will have

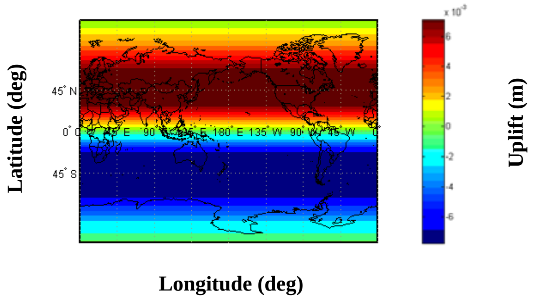

Figure 5. The uplifts due to the solid tide on an inelastic Earth (frequency dependence correction).

The same can be said for the inelastic case, once more the maximum occurs in the latitudes of about 45 degrees, the minimum effect on the -45 degrees of latitudes and zero uplift can be seen on the poles and equator. In order to see more details, it is better to investigate the difference between the two uplifts computed by regarding the Earth as either an elastic or inelastic body. The following figure shows these differences on the Earth.

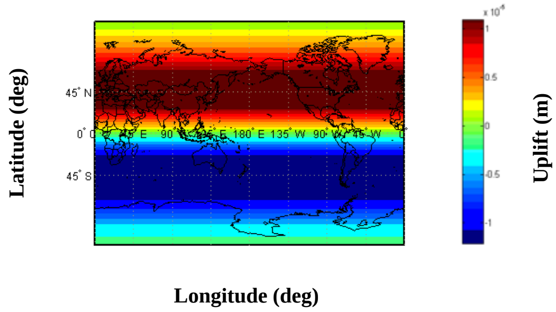

Figure 6. Difference between uplifts due to the solid Earth tide in an elastic and inelastic Earth (frequency dependence corrections).

The magnitude of the uplift differences is so small (less than a millimeter) that one can say there is no significant differences for frequency dependent components in both cases in which the Earth is regarded as elastic or not. The maximum differences can be seen in the regions with the latitude of about 45 degrees, the their minimums on the -45 latitude. The uplifts are very small on the poles and the equator.

The total variations in geopotential coefficients can be obtained by adding two frequency dependence and frequency independence effects. According to the above figures it is clear that the effects of frequency independence effects, which depend on the solar and lunar ephemeris, have greater influence than frequency dependence effects of about 100 times. In the frequency independence case the effects are related to the frequency independence nominal values of second Love number but in the frequency dependence case the variations of geopotential coefficients are due to the tidal constituents and amplitude and phase of this constituent. For considering the solid tide effects on an elastic Earth the second Love number is regarded as a real number, but in an inelastic Earth the inelasticity of the mantle cause that the second Love number to become a complex number with a small imaginary part. In both situations (frequency dependence and frequency independence), when the Earth is considered as an elastic body the Love number is regarded as a real number and if the Earth is considered as an inelastic body then the Love number must be considered as a complex number. Finally, in the frequency dependence case the time and tidal constituents have a major role. According to the figures it is clear that the frequency dependence influences have a major value in the region above the equator, in other words the northern hemisphere.

8 CONCLUSIONS AND RECOMMENDATIONS

For many aspects in Geodesy and Geophysics an elastic Earth model can easily be considered for solid tide considerations, but for more precise applications it is recommended to consider the Earth as an inelastic body. The results show that the differences between uplifts due to the solid tide computed via an elastic and inelastic body are very small so that in some applications the inelasticity of the Earth can be neglected. The greatest difference between the uplifts computed by using elastic and inelastic Earth model is about 3 mm on the poles and 2.5 mm on equator. It means that the ellipticity of the Earth is increased when an inelastic Earth model is used; and the Earth is contracted on poles and expanded on equator more. The frequency dependence corrections have the most influence on the equator and the least on the poles. The zero uplift occurs on the latitude of +45 and -45 degree, it means that the flattening of the Earth increases in both cases. For very precise applications it is recommended to use higher degree and order frequency independent Love numbers. When an elastic Earth is considered, no frequency dependence corrections are needed for the zonal tidal band. For the semi diurnal band the corrections are considered only on the real part in both the elastic and inelastic cases, but for the elasticity and inelasticity of the Earth the frequency dependence correction must be computed separately. The total effect is the sum of the frequency independence and frequency dependence effects. In conclusion one can see that the inelastic Earth is more deformable with respect to the solid tide and the ellipticity of the Earth will be increased more.

ACKOWLEDGEMENT

The author would like to express his gratitude to all of the editors and reviewers.

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