

## Fuzzy Partitioning of Radon Domain for Estimation of Water Reverberation Energy

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### Abstract

The radon transform has a wide application in seismic processing for each project in different areas. Multiple attenuation is mostly summarized in the use of radon analysis in practice, especially in marine data processing. The definition of mute function is the major challenge in parabolic radon transform. In this paper, a method for segmentation of the radon transform by fuzzy inference system is introduced to separate energy parts in the radon domain. We applied a fuzzy inference system based on the property of energy distribution and its attribute in the radon domain. The result of clustering is the partitioning of the radon domain in three major classes: 1- random noise, 2- multiple, and 3- primary and multiple. The result of applying the new method on real data has shown the applicability of the new method for separation of multiple class from other classes that can assist the processor to define the mute function in the absence of other events in the radon domain.

**Keywords:** Fuzzy inference system, Multiple attenuation, Radon transform, Fuzzy partitioning, Fuzzy C-Mean clustering.

### 1. Introduction

The Radon Transform (RT) as a strong tool has widely used in the numerous science and engineering applications. It has an extended use in seismic processing with different basis function (linear, parabolic, hyperbolic, apex shifted versions). These applications include the attenuation of multiples (Seher, 2017), ground roll attenuation and trace interpolation (Gholami & Zand, 2018), elimination of free surface multiples (Hokstad and Sollie, 2006), random noise attenuation (Zhang et al, 2021), simultaneous source separation (Akerberg et al., 2008; Trad et al., 2012; Ibrahim and Sacchi, 2014), automatic stacking velocity estimation (Wang, 2016).

One of the main applications of the RT is multiple attenuation that is one of the key step in marine seismic processing and moveout discrimination as a property to attenuate multiples in the CDP domain.

The parabolic RT was introduced by Hampson (1986) to map different events in the CMP domain to the radon domain. Hampson exerted the parabolic radon transform to NMO-corrected gathers. Primaries in parabolic RT map to zeros move-out part and multiples map to non-zero moveout part of radon domain. To increase the efficiency of parabolic (hyperbolic) RT

and easily picking multiple (primary) areas in the RT domain, some modifications are performed on standard parabolic (hyperbolic) RT.

In the case of presenting the gap in CDP, gather interpolation and/or aperture extension are needed that must use sparse RT. Sacchi and Ulrych (1995) take the benefit of the sparse RT in the frequency domain by means of Bayes rule using a Cauchy form Probability-Density Functions (PDFs). Trad (2003) presented an extension of the standard RT as apex-shifted RT to handle the effect of variable apex locations in the offset dimension.

The main problem when using RT is identifying signal portion from the noise. Some authors try to improve RT to reach better signal to noise separation in the RT domain. In modified version of RT processor one can define mute function with higher accuracy to avoid signal attention after applying reverse RT.

Gholami and Zand (2018) presented a fast algorithm for a three-parameter RT based on shifted hyperbolas. They also used split Bregman iteration and generalized Fourier Slice Theorem for the sparse solution of the problem and the associated inverse/adjoint forward transforms that led to an efficient

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algorithm for effective decomposition of long-offset seismic data. Abedi et al. (2018, 2019) used a three-parameter RT in VTI media that is a time-independent version that can better localized different seismic events; it is also more accurate, especially in the presence of vertical heterogeneity.

Zarei and Hashemi (2019, 2021) introduced a new approach of standard RT by the name of image RT for better discrimination of multiple from primary in radon domain. They used edge attributes and additional mathematical properties in the radon domain to achieve a new section for importing to the RT operator. Using this approach, only multiple events move to the radon domain and selecting multiple areas are easier than conventional RT.

In this paper, we introduced a new approach for classification of energy in the radon domain using Fuzzy Inference System (FIS) to be able to make partitions in the radon domain for different energy parts. This method can be applied on different versions of RT transform for partitioning of radon domain. Results of applying this method on synthetic and real data have a surprised segmentation of signal and noise in radon domain.

## 2. Methodology

The main goal in all of the RT versions is the separation of signal from noise in the radon domain. The direct forward parabolic radon transform is defined as Equation (1) that is commonly used for multiple attenuation;

$$m(q, \tau) = \sum_{n=1}^N d(x_n, t = \tau + \theta x_n^2). \quad (1)$$

where  $d(x_n, t)$  is the data in the offset(x)-time (t) domain, N is the number of traces,  $\theta$  is the ray parameter or slowness and  $m(q, \tau)$  is the data in the parabolic radon domain. In the parabolic radon domain, primaries transform to the near-zero residual moveout axis and multiples transform into the higher values of the residual moveouts. Nevertheless, separating multiple and primary areas in the radon panel is very acute and usually depends on the prior knowledge of the processor.

The definition of mute function is critical when the different types of multiple exist on data or area of surveying that contains

complex geology. One major problem when working with RT is changing the mute function between different processors that affects the final result, especially the 2D vintage.

### 2-1. Fuzzy Logic and Fuzzy System

Fuzzy logic was introduced by Zadeh in 1965, in which the concept of fuzzy variables versus the crisp variables is the most important achievement. The fuzzy logic is a platform to use the linguistic variable and knowledge-based information as mathematical operations. The fuzzy logic developed in a wide range of engineering applications for control and operating systems. The application of fuzzy logic and the fuzzy system has great advantages and added value especially in the interpretation of data because there are many variables in interpretation that are dependent on the scene of interpreter and change the user.

A Fuzzy Inference System (FIS) is the process of formulating from input data to output data using fuzzy theory and is mainly based on the use of linguistic/data-based rules. In a fuzzy system, each data can belong to a group and its membership will be between zero and one. Each FIS is identified using the membership function. Membership functions have different forms, such as Gaussian, triangular, trapezoidal, sigmoid, S-shaped, Z-shaped, etc. Each FIS contains three main parts: 1- Fuzzifier, 2-Inference machine and 3-Defuzzifier, and based on these properties two main types of FIS are introduced: Mamdani FIS (Mamdani and Assilian, 1975) and Sugeno FIS (Takagi and Sugeno, 1985; Sugeno and Kang, 1988).

Fuzzy system and fuzzy logic have a wide range of application in seismic data processing and interpretation. Aminzadeh and Wilkinson (2004) reviewed the application of neural network and fuzzy logic in seismic object detection. They focused on a rule-based neural network to combine different seismic attributes and effectively bringing data with interpreter's knowledge to reduce the exploration risk.

Hashemi et al. (2008) presented a new technique based on the unsupervised clustering with a fuzzy GK clustering algorithm to detect the seismic random noise in pre and post stack data. They used an

adaptive distance norm to discover centers of ellipsoidal clusters and create a partition matrix which defines the soft decision boundaries between seismic events and random noise.

Hadiloo et al. (2018) compared and evaluated the capability of two unsupervised methods, Fuzzy c-means (FCM) and Gustafson Kessel (GK) and one supervised method, Adaptive Neuro-Fuzzy Inference Systems (ANFIS) in revealing the presence of a channel system.

Chongjin et al. (2020) used the guided FCM clustering method for joint inversion of three different physical properties of rocks (gravity, magnetic, seismic) of the subsurface model. In this approach, he received clustering results that were steady with the geophysical properties and achieved more reliable results after interpretation.

In below, we will review the Mamdani FIS and FCM clustering's that are used for generating FIS for the classification of energy in the radon domain.

## 2-2. Fuzzy C-Means Clustering

FCM clustering (Dunn, 1973) as unsupervised clustering have wide application in seismic interpretation and processing. The fuzzy c-mean clustering is an unsupervised clustering algorithm that allows us to build a fuzzy partition from data. The algorithm depends on a "c" parameter (2) which corresponds to the degree of fuzziness of the solution. Large values of "c" will become less sharp in the classes and all elements are inclined to be the property of all clusters. The solutions of the optimization problem are controlled by the parameter "c". That is, different selections of "c" will typically eventuate to different partitions.

Let  $\mathbf{X} = \{x_1, \dots, x_n\}$  be a set of given data, where each data point  $x_k$  ( $k = 1, 2, \dots, n$ ) is a vector in  $\mathbf{R}^p$ ,  $\mathbf{U}_{cn}$  be a set of real  $c \times n$  matrices, and c be an integer,  $2 \leq c < n$ , then, the fuzzy c-partition space for  $\mathbf{X}$  is the set:

$$\mathbf{M}_{fcn} = \{\mathbf{U} \in \mathbf{U}_{cn} : u_{ik} \in [0, 1], \sum_{i=1}^c u_{ik} = 1, 0 < \sum_{k=1}^n u_{ik} < n\} \quad (2)$$

where  $u_{ik}$  is the membership value of  $x_k$  in

cluster  $i$  ( $i = 1, \dots, c$ ).

The aim of the FCM algorithm is to discover an optimum fuzzy c-partition and corresponding prototypes minimizing the objective function:

$$J(\mathbf{U}, \mathbf{V}; \mathbf{X}) = \sum_{k=1}^n \sum_{i=1}^c (u_{ik})^m \|x_k - v_i\|^2 \quad (3)$$

where  $\mathbf{V} = (v_1, v_2, \dots, v_c)$  is a matrix of unknown cluster centers (prototypes)  $v_i \in \mathbf{R}^p$ ,  $\|\cdot\|$  is the Euclidean norm, and the weighting exponent  $m$  in  $[1, \infty)$  is a constant that affects the membership values.

To minimize the rule  $J$  and under the fuzzy constraints defined in Equation (2), the FCM algorithm is defined as an intermittent minimization algorithm as follows. Choose values for  $c, m$ , and  $\varepsilon$ , a small positive constant; then, produce randomly a fuzzy c-partition  $U^0$  and set iteration number  $t = 0$ . A two-step iterative progress works as follows. Given the membership values  $u_{ik}^{(t)}$ , the cluster centers  $v_i^{(t)}$  ( $i = 1, \dots, c$ ) are calculated by:

$$v_i^{(t)} = \frac{\sum_{k=1}^n (u_{ik}^{(t)})^m x_k}{\sum_{k=1}^n (u_{ik}^{(t)})^m} \quad (4)$$

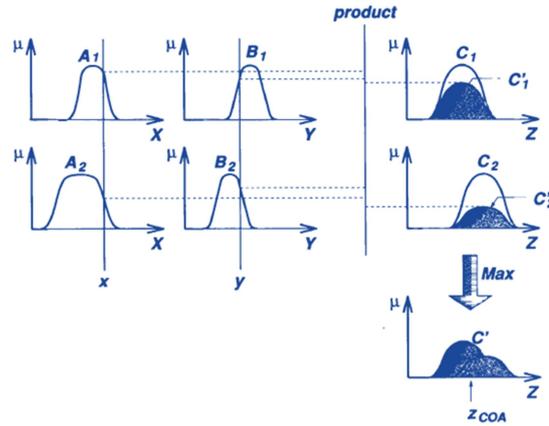
Given the new cluster centers  $v_i^{(t)}$ , update membership values  $u_{ik}^{(t)}$

$$u_{ik}^{(t+1)} = \left[ \sum_{j=1}^c \left( \frac{\|x_k - v_i^{(t)}\|^2}{\|x_k - v_j^{(t)}\|^2} \right)^{\frac{2}{m-1}} \right]^{-1} \quad (5)$$

The process stops when  $|U^{t+1} - U^t| < \varepsilon$ , or a predefined number of iterations is touched.

## 2-3. Mamdani FIS

The most important part of fuzzy inference methods is the type of membership function (MF) and the method of their implementation. In the Mamdani FIS method, the output of the fuzzy function is a fuzzy system. After the aggregation process, a fuzzy return system is used for each output variable. In the Mamdani method according to Figure 1, the minimum operator is used (Mamdani and Assilian, 1975).



**Figure 1.** Graphic representation of Mamdani fuzzy inference method using multiplication and maximum operator (Jang et al., 1997).

In this study, we will use FCM clustering as an initial clustering to achieve Mamdani based fuzzy system to divided energy on radon domain to different parts.

**3. Application**

For initial clustering using FCM, we used Image radon transform (Zarei and Hashemi, 2019) of input data and entropy as an attribute for accurate clustering.

The number of rules is critical and must be greater than the number of clusters that exist on data. The number of clusters in the radon domain is 3 that contain random noise, primary and multiple events. Therefore, the number of clusters for FCM clustering must be greater than 3 and after testing, we chose 10 clusters for FCM Clustering.

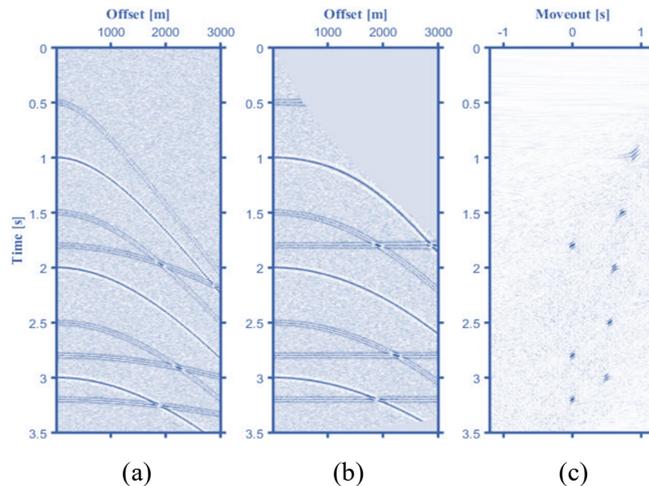
We applied this method on synthetic and real data with different range of noise to show the ability of this method to partition radon domain to the different energy parts. A synthetic CMP gather is generated using a Matlab code. It contains primary and

multiple events. The model features are listed in Table 1. The sample rate is 2 (ms) and the offset variety is 12.5 (m) to 3025 (m). A zero-phased Ricker wavelet with central frequency 20 (Hz) is used to generate a seismogram. The data, its corresponding NMO corrected gather (of input CMP gathers) and its parabolic radon transform are shown in Figure 2.

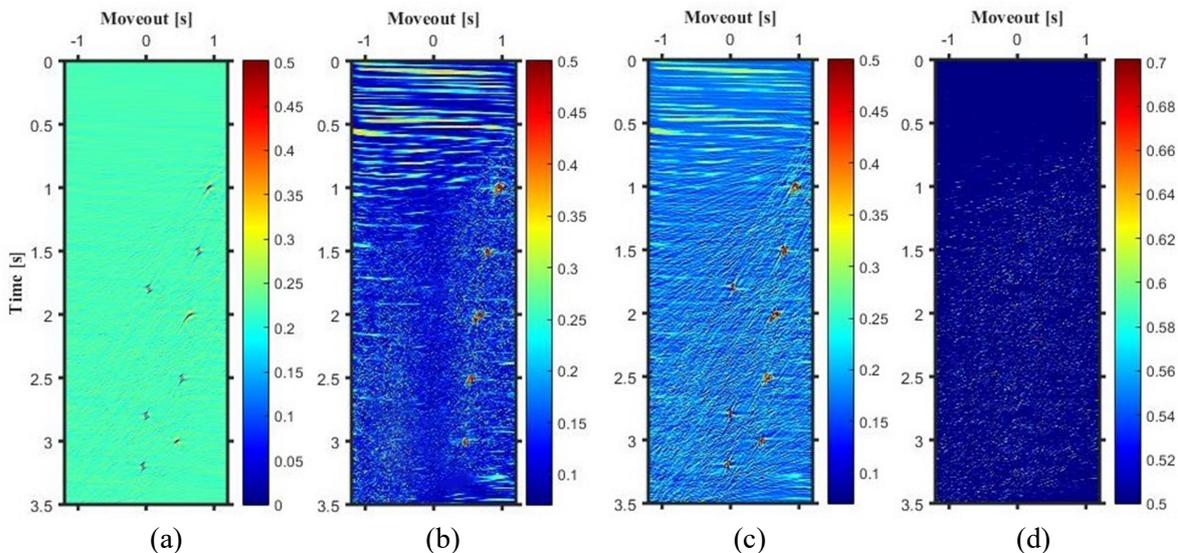
**Table 1.** The properties of synthetic model.

Event No.	TWT (ms)	Velocity (m/s)
1	500	1500
2	1800	2400
3	2800	2800
4	3200	3200

After applying the presented method on synthetic data as shown in Figure 3, the energy distribution in RT domain well define and multiple parts have been separated as a single class.



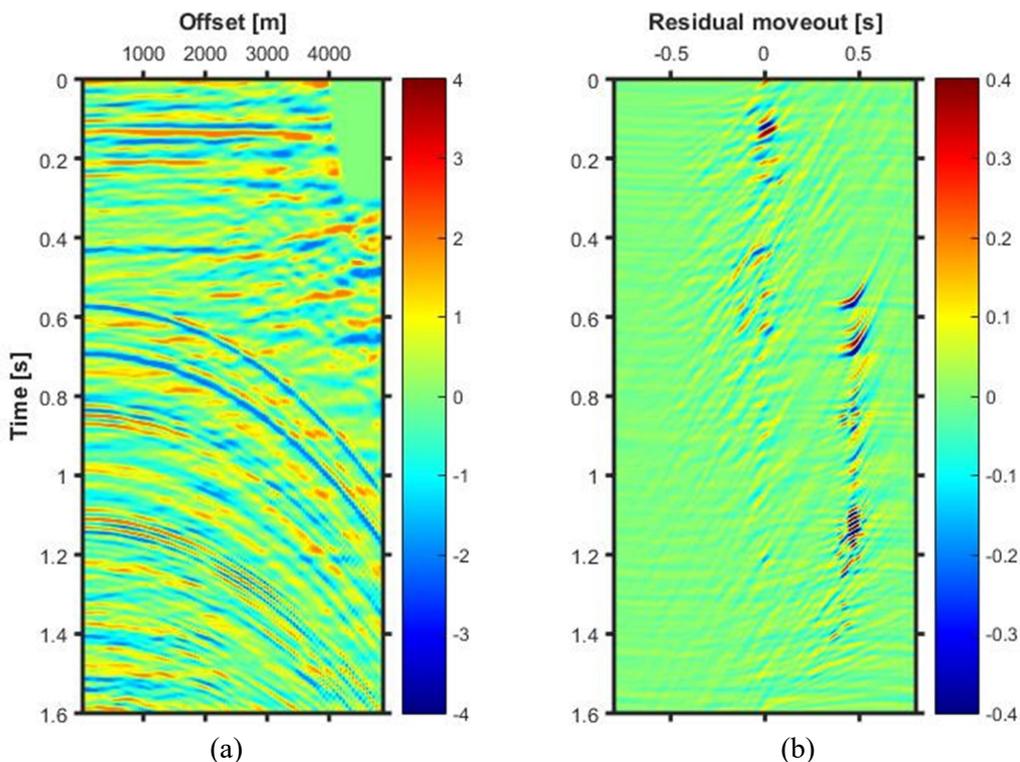
**Figure 2.** Synthetic data (a), NMO Gather (b) and Radon Transform of NMO gather (c).



**Figure 3.** (a) Initial RT, (b) the class including the multiple parts, (c) the class including primary and multiple and (d) the class including random noise. The color bar presents the amplitudes.

Another example for testing this method is data from Gulf of Mexico. Figure 4 shows NMO gather from the Gulf of Mexico that contains multiples and its parabolic RT. The input of FCM clustering is the radon transform of data shown in Figure 4 and some attributes that are extracted from radon panel. As we expected, the output of this clustering contains 10 clusters for which

each cluster consists of different parts of the radon panel. There are some classes that include primary and multiple simultaneously and some classes contained random noise, and finally, there is some class that only contains multiple. In all clusters, there is some random energy that is no problem for the definition of the target area.



**Figure 4.** NMO Corrected Gather from Gulf of Mexico (a) and its parabolic radon transform (b). The color bar represents the amplitudes.

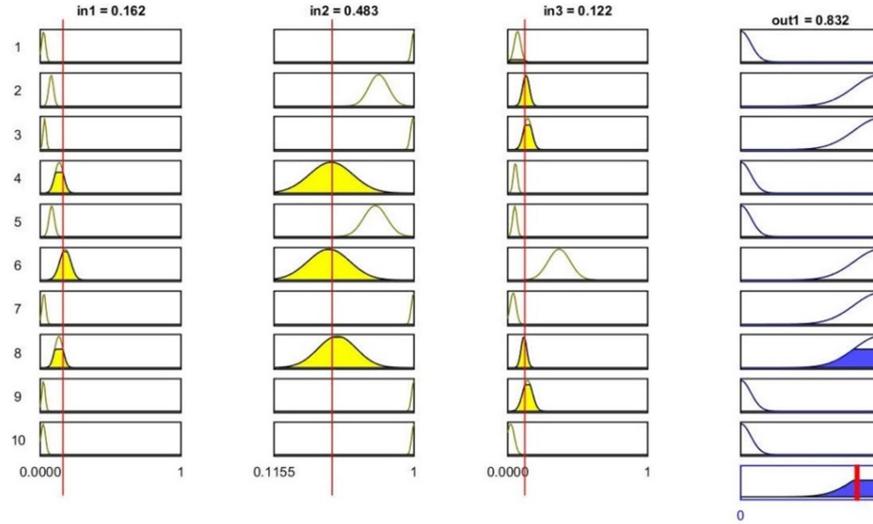


Figure 5. The fuzzy rule that extracted from data based on energy distribution in the radon domain.

Figure 5 shows the fuzzy rule obtained after clustering using FCM clustering. There are three main classes, some of which are the redundant rules. This redundant rules can be reduced and the accuracy of clustering will be increased.

The different output of FIS shown in Figure 6 including three different classes exist on radon domain. Using this approach to segment radon transform processor can easily define the mute function for multiple attenuations in the absence of the primary event.

4. Conclusions

In this paper, we review the parabolic radon transform and its limitation to separation of the multiple energy from primary in the radon domain. It also used the advantage of the fuzzy system

to the partitioning of radon domain. A Fuzzy system is a tool for managing data based on data properties and can be used for segmentation of radon domain for signal and noise parts to help the processor for the definition of the mute function.

A new FIS are generated based on radon transform and some attributes are extracted in the radon domain as an input for FIS. The output of the new FIS included three main classes: 1- random noise, 2- multiples, and 3- primary and multiple.

The result of applying this new method to synthetic and real data shows that the segmentation of radon domain has a great benefit to define the mute function in absence of the primary event. Thus multiple events can be modeled easily and can be subtracted from the original data.

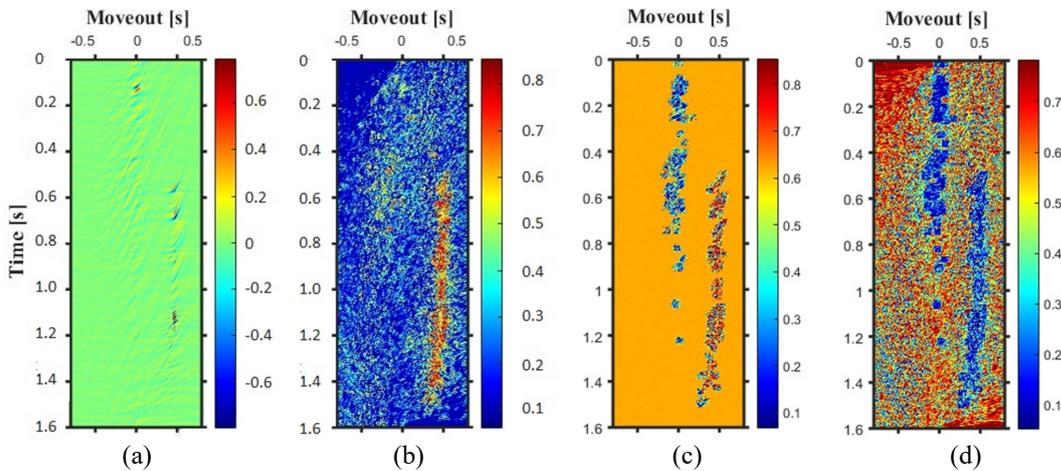


Figure 6. Initial radon transform (a), FIS output cluster including multiple energy (b), FIS output cluster including multiple and primary energy (c) and FIS output cluster including random noise (d). The color bar represents the magnitude of attributes.

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