University of Tehran Press
Journal of the Earth and Space Physics
Homepage: https://jesphys.ut.ac.ir
Vol. 48, No. 4, Winter 2023, P. 99-106
(Research Article)

Modelling Thermal Convection of Earth Mantle with Aspect Code

Asaadi, N. [1] | Norouzi, S. [2]

1. **Corresponding Author**, Department of Physics, Faculty of Science, University of Zanjan, Zanjan, Iran. E-mail: nazilaasaadi@znu.ac.ir
2. Department of Physics, Faculty of Science, University of Zanjan, Zanjan, Iran. E-mail: samane.norouzi7@gmail.com

(Received: 12 Feb 2022, Revised: 28 Aug 2022, Accepted: 4 Oct 2022, Published online: 5 March 2023)

Abstract
The study of mantle convection is one of the most important topics in geodynamics. Mantle convection causes the transfer of internal heat to the cold parts of the Earth, and the effects of this heat transfer are observed as the motion of tectonic plates on the Earth's surface. Earthquakes, volcanism, and mountain building at the plate margins result from the movement of tectonic plates. Although the mantle occupies a large volume of the Earth, there are many fundamental questions about mantle composition, rheology, dynamics, and history. Many of these questions remain unanswered due to our indirect observations of the mantle. A major tool to study mantle dynamics is numerically analyzing mantle convection equations. In this work, we used Aspect -short for Advanced Solver for Problems in Earth's Convection- code to simulate mantle convection. The geometric model used in the simulation is a box of 4200 km by 3000 km. Using this code, we investigated the effect of different Rayleigh numbers on controlling the mantle convection and creating mantle plumes. Results show that the number of mantle plumes increases with increasing Rayleigh number, and the rising mantle plumes become thinner with the Rayleigh number increasing. Finally, we studied the relationship between the Rayleigh number and the Nusselt number (surface heat flux). We conclude that there is a power-law relation between Rayleigh and Nusselt numbers.

Keywords: Mantle convection, Numerical simulation, Mantle plume, Rayleigh number, Nusselt number.

1. Introduction
Located 150 million km from the Sun, Earth is the third planet in the solar system, a dynamic planet that is constantly changing. The Earth's interior is divided into three layers: the crust, the mantle and the core. The oceanic crust, about 6 km thick, has a basaltic composition. The continental crust has a silicic composition with an average thickness of 30 km. The mantle has an ultrabasic composition; the boundary between the mantle and the crust is called the Moho seismic discontinuity. The mantle has seismic discontinuities at about 410 km and 660 km. The average mantle density is between 3.3 and 4.8 g cm^{-3}. The core is almost iron, the outer core is liquid, and the inner core is solid (Schubert et al., 2004).
In the late 1930s, Arthur Holmes proposed that thermal convection in the Earth's mantle provided the necessary force for continental drift (Holmes, 1931). The main hypothesis of plate tectonics was formulated by Morgan (1968). The surface expression of mantle convection is plate tectonics. Although the Earth's mantle behaves like an elastic solid on short-term scales, it acts like a highly viscous fluid (10^{21} Pa.s) on long-term scales. Therefore, we consider the Earth's mantle a fluid with high viscosity to investigate heat convection (Schubert et al., 2004).
Numerical analysis is a fundamental tool for understanding mantle convection (Davies, 2004) and has a rich history since the late 1960s (Zhong et al., 2007). The first 2D mantle thermal convection models were discussed by Torrance and Turcotte (1971) with temperature-dependent viscosity and another two-dimensional mantle thermal convection (Richter, 1973; Moore and Weiss, 1973; Houston and DeBremacker, 1975; Parmentier and Turcotte, 1978; Lux et al., 1979; Schubert and Zebib, 1980). The first three-dimensional spherical mantle thermal convection models were investigated by (Baumgardner, 1985; Machetel et al., 1986) and cartesian models by (Cserepes et al.,

Cite this article: Asaadi, N., & Norouzi, S. (2023). Modelling Thermal Convection of Earth Mantle with Aspect Code. *Journal of the Earth and Space Physics*, 48(4), 99-106. DOI: http//doi.org/10.22059/jesphys.2022.338634.1007402

Publisher: University of Tehran Press.
DOI: http//doi.org/10.22059/jesphys.2022.338634.1007402
Print ISSN: 2538-371X
Online ISSN: 2538-3906

1988; Houseman, 1988), and another three-dimensional spherical mantle convection (Baumgardner, 1988; Glatzmaier, 1988; Glatzmaier et al., 1990; Bercovici et al., 1989a, 1989b, 1989c, 1991a, 1991b; Schubert et al., 1990; Zhong et al., 2008; Tackley, 2008), and cartesian mantle convection papers (Travis et al., 1990a, 1990b; Cserepes and Christensen, 1990; Ogawa et al., 1991; Christensen and Harder, 1991). Nowadays, there are powerful computer codes for simulating 3D spherical mantle convection (Zhong et al., 2000; Tackley, 2008; Heister et al., 2017; Kronbichler et al., 2012).

There are various methods for numerically solving the governing equations of fluid dynamics, including finite difference, finite element, and finite volume methods. In the finite difference method, the derivatives of the functions and values of the differential equations are approximated using the Taylor expansion. The main limitation of this method is the inability to make non-rectangular discretization. The desired domain is networked into regular small triangular squares in the finite element method. This method is more efficient than other methods and has high accuracy. In the finite volume method, the desired domain is divided into several volumes or cells (Gerya, 2010). In this paper, we used the Aspect code (Bangerth et al., 2017), based on the finite element method, to solve the thermal convection dynamics of the Earth's mantle.

Mantle plumes are concentrated, nearly cylindrical upward flows of hot mantle material, showing a rise in the convecting mantle (Bercovici et al., 1989a). Mantle plumes have been the field of study in various previous numerical modeling studies (Farnetani and Richards, 1995; Trompert et al., 1998a, 1998b; d'Acremont et al., 2003; Zhong, 2005; Lin and van Keken, 2005; Sobolev et al., 2011; Ballmer et al., 2013; Trubitsyn et al., 2018).

The important non-dimensional numbers for mantle convection are the Rayleigh number and the Nusselt number. The Rayleigh number describes the relationship between buoyancy and viscosity within the fluid and gives a sense of the vigour of convection. The Nusselt number is a ratio of convective heat transfer to conductive heat transfer. Previous studies show the power-law relationship between Rayleigh and Nusselt numbers (Turcotte et al., 1967; Hansen & Ebel, 1984; Jarvis, 1984; Olsen, 1987; Korenaga, 2003; Wolstencroft et al., 2009).

In this paper, we used the Aspect code to simulate mantle convection. Using results from mantle convection simulations with Aspect code, we investigated the relationship between the number of mantle plumes and the Rayleigh number. Then we studied the power-law relationship between Ra and Nu. Results show that there is a close agreement with previous studies.

Section 2 presents equations governing mantle convection dynamics, Rayleigh number and Nusselt number. Model set-up is described in section 3. Our results and conclusions are presented in section 4 and section 5, respectively.

2. Equations Governing Mantle Convection Dynamics

We discuss the fundamental equations of mantle convection, and consider the conservation of mass (1), momentum (2), and energy for a fluid continuum (3) (Schubert et al., 2004). However, the mantle is composed of solid rocks but deforms as fluid on geophysical time scales. Therefore, we can study mantle convection dynamics with conservation equations as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (1)$$

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i \quad (2)$$

$$\rho T \frac{Ds}{Dt} = \tau_{ij} \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_i}\left(k_B \frac{\partial T}{\partial x_i}\right) + \rho H \quad (3)$$

where ρ is the fluid density, u_i is the fluid velocity, τ_{ij} is the stress tensor deviator, k_B is the thermal conductivity, and H is the rate of internal heat production per unit mass. The rheological law between deviatoric stress, τ_{ij} and strain rate, e_{ij} is:

$$\tau_{ij} = 2\mu e_{ij} - \frac{2}{3}\mu e_{kk}\delta_{ij} = \mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\mu \frac{\partial u_k}{\partial x_k}\delta_{ij}) \quad (4)$$

where μ is viscosity.

2-1. Rayleigh Number

The Rayleigh number controls the vigour of mantle convection (Ismail-Zadeh & Tackley, 2010). Substitution of (4) into (2) gives the Navier-Stokes equation:

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right)\right] + \rho g_i \quad (5)$$

The dimensionless form of the Navier-Stokes equation is:

$$\frac{\rho\kappa}{\mu}\frac{Du}{Dt} = \nabla^2 u - \nabla p - \frac{\alpha\Delta T\rho_0 g h^3}{\mu\kappa}T \quad (6)$$

$$Ra = \frac{\alpha\Delta T\rho_0 g h^3}{\mu\kappa} \quad (7)$$

where Ra is the Rayleigh number (Schubert et al., 2004). We will investigate the dependence of mantle convection on the Rayleigh number in the next section.

2-2 Nusselt Number

The Nusselt number is defined as the ratio of the convective heat flow $q = \frac{Q}{L}$ to the conduction heat flow $q_k = \frac{k\Delta T}{D}$, where Q is heat fluxes through the top boundary. The Nusselt number directly measures the efficiency of convection as a heat transport mechanism versus conduction. The law of heat transfer is defined as a power-law relation in terms of the Nusselt number as follows (Schubert et al., 2004):

$$Nu = cRa^{\beta} \quad (8)$$

where c and β are constant coefficients. We will discuss these coefficients in the results section.

3. Model Setup

We used the Aspect code that solves the conservation equations for mass, momentum, and energy in the Boussinesq approximation (the reference temperature and the reference density are constant). We consider a closed box with an aspect ratio of 1.4, leading to a box width and depth of 4200 km and 3000 km, respectively. The top temperature is fixed to 273 K, and the bottom is set to 3600 K. The parameter values used in the Aspect code are defined in Table 1. We used different Rayleigh numbers to investigate mantle convection patterns. These numbers with their viscosity are shown in Table 2.

4. Results

We simulate mantle convection with aspect code for different Rayleigh numbers. The results are shown in Figure 1. The Rayleigh number shows the presence and strength of convection in the mantle. First, we examined the Rayleigh Number 5×10^4. The mantle convection pattern is seen because the Rayleigh number is larger than the critical Rayleigh number. However, mantle plumes do not form. The mantle plume is an ascending stream of thermal convection with a mushroom-like shape and a limited lifespan. For the Rayleigh numbers smaller than the critical Rayleigh number, heat transfer occurs by conduction, and no thermal convection is observed. By increasing the Rayleigh number to 5×10^5, mantle plumes begin to form, and by increasing the Rayleigh number to 5×10^7, we can see an increase in the number of mantle plumes and a thinning of the ascending plumes, Figure 1 (d). These results are in good agreement with the results of others (Trompert et al., 1998 a, 1998b; Zhong, 2005; Trubitsyn et al., 2018). Finally, we investigate the relation between the Rayleigh and Nusselt numbers. There is a power-law relation between Rayleigh and Nusselt numbers, Figure 2. Figure 2 shows the values obtained for c and β equal 0.28 and 0.31, respectively. Table 3 compares the β quantity in Equation (8) with previous calculations. Based on these results, the values obtained in this study are in close agreement with previous studies.

Table 1. Parameter values.

Parameter	Symbol	Value
Mantle density	ρ	3300 kg m^{-3}
Thermal conductivity	k	4.7 m^{-1} K^{-1}
Thermal diffusivity	κ	$1 \cdot 1394 \times 10^{-6}$ m^2 s^{-1}
Gravity acceleration	g	9.8 m s^{-2}
Thermal expansion coefficient	α	2×10^{-5} K^{-1}
Heat capacity	c_p	$1 \cdot 25 \times 10^3$ J mol^{-1}

Table 2. Rayleigh number and Viscosity Pa.s

Rayleigh number	Viscosity (Pa.s)
5×10^4	$1 \cdot 02 \times 10^{24}$
5×10^5	$1 \cdot 02 \times 10^{23}$
5×10^6	$1 \cdot 02 \times 10^{22}$
5×10^7	$1 \cdot 02 \times 10^{21}$

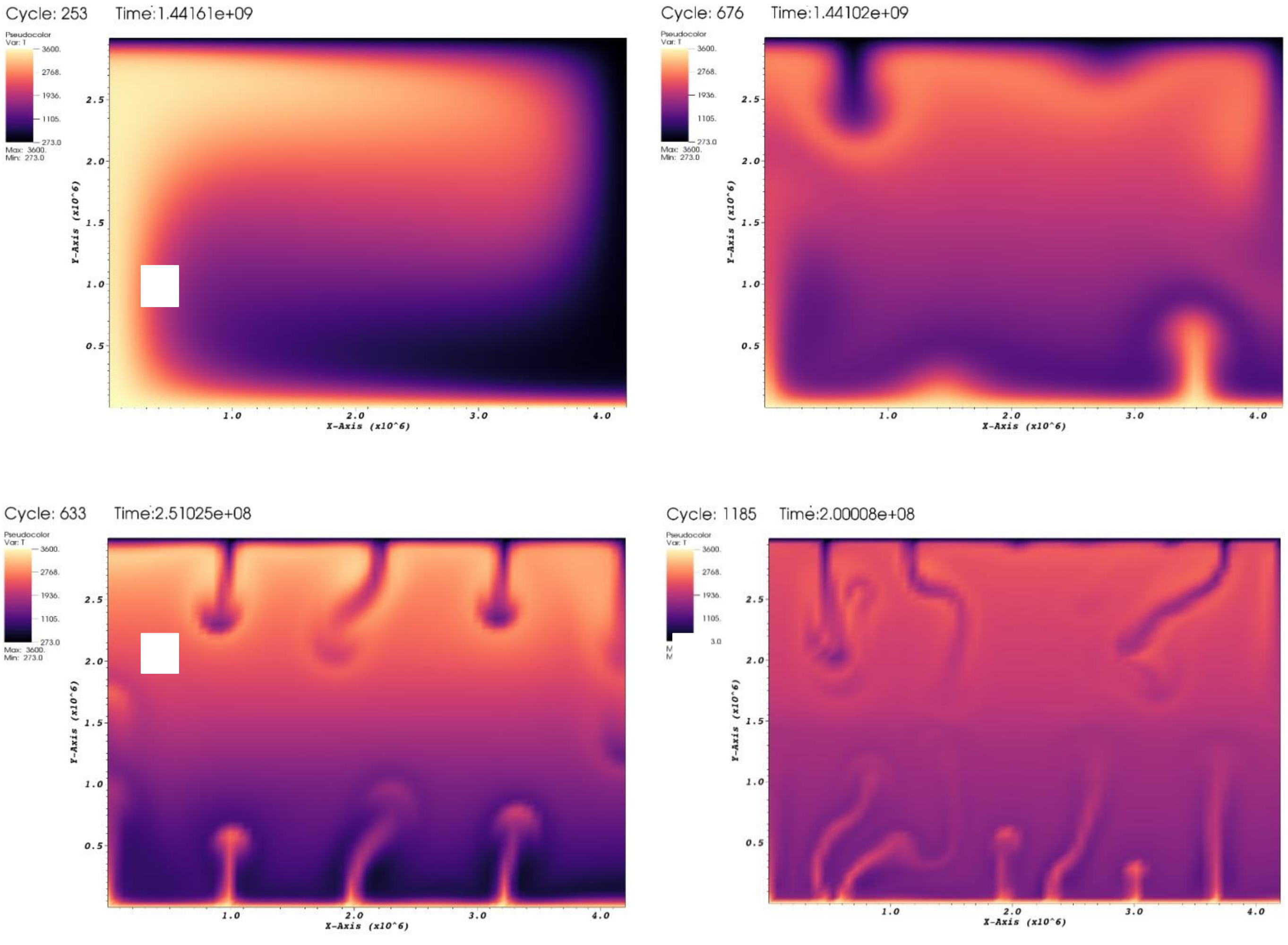

Figure 1. a) The Rayleigh number is 5×10^4 and viscosity is $1 \cdot 02 \times 10^{24}$ Pa.s. b) The Rayleigh number is 5×10^5 and viscosity is $1 \cdot 02 \times 10^{23}$ Pa.s. c) The Rayleigh number is 5×10^6 and viscosity is $1 \cdot 02 \times 10^{22}$ Pa.s. d) The Rayleigh number is 5×10^7 and viscosity is $1 \cdot 02 \times 10^{21}$ Pa.s.

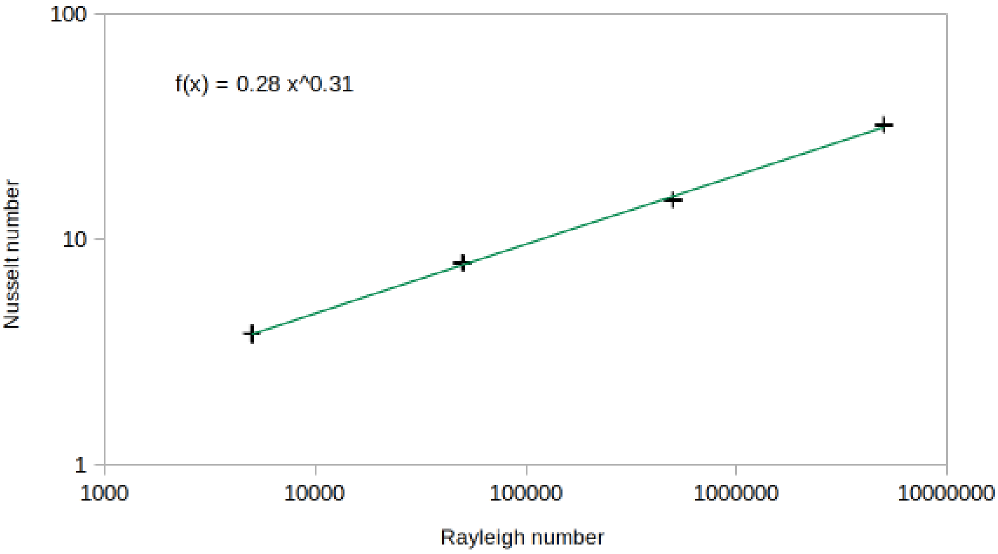

Figure 2. The Nusselt number as a function of the Rayleigh number on logarithmic scales.

Table 3. Comparison of β quantity.

β	study
0.33	Turcotte et al., 1967
0.34	Hansen et al., 1984
0.318	Jarvis, 1984
0.33	Olsen, 1987
0.3	Korenaga, 2003
0.29	Wolstencroft et al., 2009
0.31	This study

5. Discussion and conclusion

The thermal convection of the mantle plays a significant role in the movements of tectonic plates and geophysical phenomena such as earthquakes, volcanoes, etc. Therefore, the study of mantle convection is of great importance. To study mantle convection, we considered the mantle to be a constant high-viscosity fluid. We used the Aspect code based on the finite element method to solve the equations governing mantle convection. In the Earth's mantle, the Rayleigh number is estimated to be 5×10^6 to 5×10^8. The Rayleigh number is an important parameter that controls the nature of convection. We studied the role of the Rayleigh numbers in convective patterns. Conclusions indicate that for small values of Ra a system will not convect and heat will be transported only by conduction. For values of Ra greater than critical value (87<Ra_c<1100) convection will be present in the system. The results showed that by increasing the Rayleigh number, mantle plumes are created, and increasing the Rayleigh number increases the number of plumes and makes the ascending plumes thinner. Finally, by examining the relationship between the Rayleigh and the Nusselt numbers, we obtained the power-law relationship between them. We find that the Nusselt number is proportional to the Rayleigh number to the 0.31 power that has a good agreement with previous studies.

References

Ballmer, M.D., Ito, G., Wolfe, C.J., & Solomon, S.C. (2013). Double layering of a thermochemical plume in the upper mantle beneath Hawaii. *Earth and Planetary Science Letters*, 376, 155–164.

Bangerth, W., Dannberg, J., Gassmoller, R., & Heister, T. (2017). *ASPECT: Advanced Solver for Problems in Earth's Convection*, CIG.

Baumgardner, J.R. (1985). Three-dimensional treatment of convective flow in the Earth's mantle. *J. Statis. Phys.*, 39, 501-11.

Baumgardner, J.R. (1988). Application of supercomputers to 3-d mantle convection. In: *Physics of the Planets*, ed. S.K. Runcorn, pp. 199-231, New York: Wiley.

Bercovici, D., Schubert, G., Glatzmaier, G.A., & Zebib, A. (1989a). Three-dimensional thermal convection in a spherical shell. *J. Fluid Mech.*, 206, 75-104.

Bercovici, D., Schubert, G., & Glatzmaier, G.A. (1989b). Three-dimensional, spherical models of convection in the Earth's mantle. *Science*, 244, 950-55.

Bercovici, D., Schubert, G., & Glatzmaier, G.A. (1989c). Influence of heating mode on three-dimensional mantle convection. *Geophys. Res. Lett.*, 16, 617-20.

Bercovici, D., Schubert, G., & Glatzmaier, G.A. (1991a). Three-dimensional convection of an infinite Prandtl number compressible fluid in a basally heated spherical shell. 1. *Fluid Mech*. 239, 683-719.

Bercovici, D., Schnbert, G., & Glatzmaier, G.A. (1991b). Modal growth and coupling in three-dimensional spherical convection. *Geophys. Astrophys. Fluid Dyn.*, 61, 149-159.

Christensen, U., & Harder, H. (1991). Three dimensional convection with variable viscosity. *Geophys. J. Int.*, 104, 213-26.

Cserepes, L., Rabinowicz, M. and Rosemberg-Borot, C. (1988), Three-dimensional infinite Prandtl number convection in one and two layers and implications for the Earth’s gravity field. *Journal of Geophysical Research*, 93, 12009–25.

Cserepes, L., & Christensen, U. (1990), Three dimensional convection under drifting plates. *Geophys. Res. Lett.*, 17, 1497-1500.

d’Acremont, E., Leroy, S., & Burov, E.B. (2003). Numerical modelling of a mantle plume: the plume head–lithosphere interaction in the formation of an oceanic large igneous province. *Earth and Planetary Science Letters*, 206(3), 379–396.

Davies, G.F. (2004). *Dynamic Earth: Plates, Plumes and Mantle Convection*. Cambridge Univ. Press, Cambridge.

Farnetani, D.G., & Richards, M.A. (1995). Thermal entrainment and melting in mantle plumes. *Earth and Planetary Science Letters*, 136(3), 251–267.

Gerya, T. (2010). *Introduction to Numerical Geodynamic Modelling*. Cambridge Univ. Press, Cambridge.

Glatzmaier, G.A. (1988). Numerical simulations of mantle convection: Time-dependent, three- dimensional, compressible spherical shell. *Geophys. Astrophys. Fluid Dyn.*, 43, 223-64.

Glatzmaier, G.A., Schubert, G., & Bercovici, D. (1990). Chaotic, subduction-like down flows in a spherical model of convection in the Earth's mantle. *Nature*, 347, 274-77.

Hansen, U.,& Ebel, A. (1984). Experiments with a numerical model related to mantle convection: boundary layer behavior of small and large scale flows. *Phys. Earth Planet*, 374-390.

Heister, T., Dannberg, J., Gassmöller, R., & Bangerth, W. (2017). High accuracy mantle convection simulation through modern numerical methods II: realistic models and problems. *Geophys. J. Int.*, 210(2), 833–851, doi:10.1093/gji/ggx195.

Holmes, A. (1931). Radioactivity and Earth movements. *The Geological Society of Glasgow*, 18, 559–606.

Houseman, G. (1988). The dependence of convection planform on mode of heating. *Nature*, 332, 346–9.

Houston, M.H. Jr., & DeBremacker, J.-Cl. (1975). Numerical models of convection in the upper mantle. *J. Geophys. Res.*, 80, 742-51.

Ismail-Zadeh, A., & Tackley, P.J. (2010). *Computational Methods for Geodynamics*. Cambridge Univ. Press, Cambridge.

Jarvis, G.T. (1984). Time-dependent convection in the Earth’s mantle. *Phys. Earth Planet*, 305-327.

Kronbichler, M., Heister, T., & Bangerth, W. (2012). High accuracy mantle convection simulation through modern numerical methods. *GJI*, 12-29.

Korenaga, J. (2003). Energetics of mantle convection and the fate of fossil heat. *Geophys. Res, Lett*., 1437.

Lin, S.-C., & van Keken, P.E. (2005). Multiple volcanic episodes of flood basalts caused.

by thermochemical mantle plumes. *Nature*, 436(7048), 250–252.

Lux, R.A., Davies, G.F., & Thomas, J.H. (1979). Moving lithospheric plates and mantle convection. *Geophys. J. R. Astron. Soc.*, 58, 209-28.

Machetel, P., Rabinowicz, M., & Bernardet, P. (1986). Three-dimensional convection in spherical shells, Geophys. *Astrophys. Fluid Dyn.*, 37, 57-84.

Moore, D.R., & Weiss, N.O. (1973). Two-dimensional Rayleigh-Bimard convection. *J. Fluid Mech.*, 58, 289-312.

Morgan, W.J. (1968). Rises, trenches, great faults, and crustal blocks. *J. Geophys. Res*., 73, 1959–82.

Ogawa, M., Schubert, G., & Zebib, A., (1991). Numerical simulations of 3-D thermal convection of a strongly temperature dependent viscosity fluid. *J. Fluid Mech.*, In press.

Olsen, P. (1987). A comparison of heat transfer laws for mantle convection at very high Rayleigh numbers. *Phys. Earth Planet*, 153-160.

Parmentier, E.M., & Turcotte, D.L. (1978). Two-dimensional mantle flow beneath a rigid accreting lithosphere. *Phys. Earth Planet. Inter.*, 17, 281-89.

Richter, F.M. (1973). Dynamical models for seafloor spreading. *Rev. Geophys. Space Phys.*, 11, 223-87.

Schubert, G., & Zebib, A. (1980). Thermal convection of an internally heated infinite Prandtl number fluid in a spherical shell. *Geophys. Astrophys. Fluid Dyn.*, I S: 65-90.

Schubert, G., Bercovici, D., & Glatzmaier, G.A. (1990). Mantle dynamics in Mars and Venus: Influence of an immobile lithosphere on three-dimensional mantle convection. *J. Geophys. Res.*, 95, 14, 105-29.

Schubert, G., Turcotte, D.L., & Olson, P. (2004). *Mantle Convection in the Earth and Planets*. Cambridge Univ. Press, Cambridge.

Sobolev, S.V., Sobolev, A.V., Kuzmin, D.V., Krivolutskaya, N., Petrunin, A.G., Arndt, N., Radko, V.A., & Vasiliev, Y.R. (2011). Linking mantle plumes, large igneous provinces and environmental Catastrophes *Nature*, 477(7364), 312–316.

Tackley, P.J. (2008). Modelling compressible mantle convection with large viscosity contrasts in a three-dimensional spherical shell using the yin-yang grid. *Phys. Earth Planet. Inter.*, 171(1–4), 7–18.

Torrance, K.E., & Turcotte, D.L. (1971). Thermal convection with large viscosity Variations. *Journal of Fluid Mechanics*, 47, 113–25.

Travis, B., Olson, P., & Schubert, G. (1990a). The transition from two-dimensional to three-dimensional planforms in Infinite Prandtl number thermal convection. *J. Fluid Mech.*, 216, 71-91.

Travis, B., Weinstein, S., & Olson, P. (1990b), Three-dimensional convection planforms with internal heat generation. *Geophys. Res. Lett.*, 17, 243-46.

Trompert, R.A., & Hansen, U. (1998a). On the Rayleigh number dependence of convection with a strongly temperature-dependent viscosity. *Phys. Fluids*, 10, 351–360.

Trompert, R., & Hansen, U. (1998b). Mantle convection simulations with rheologies that generate plate-like behavior. *Nature*, 686-689.

Trubitsyn, V.P., & Evseev, M.N. (2018). Plume Mode of Thermal Convection in the Earth's Mantle, Izv. Phys. *Solid Earth*, 838-848.

Turcotte, D.L., & Oxburgh, E.R. (1967). Finite element calculations of very high Rayleigh number thermal convection. *J. Fluid Mech*., 29-42.

Wolstencroft, M., Davies, D.H., & Daveis, D.R. (2009). Nusselt–Rayleigh number scaling for spherical shell Earth mantle simulation up to a Rayleigh number of 10^9. *Physics of the Earth and Planetary Interiors*, 176, 132–141.

Zhong, S.J., Zuber, M.T., Moresi, L., & Gurnis, M. (2000). Role of temperature-dependent viscosity and surface plates in spherical shell models of mantle convection. *J. Geophys. Res.*, 105(B5), 11,063–11,082.

Zhong, S. (2005). Dynamics of thermal plumes in three-dimensional isoviscous thermal convection. *GJI*, 289-300.

Zhong, S.J., Yuen, D.A., & Moresi, L.N. (2007). Numerical methods in mantle convection. In Bercovici, D. (ed.) Treatise in Geophysics, Volume 7, (editor-in-chief: Gerard Schubert), Elsevier, 227–52.

Zhong, S., McNamara, A., Tan, E., Moresi, E., & Gurnis, M. (2008). A benchmark study on mantle convection in a 3-D spherical shell using CitcomS. *Geochemistry, Geophysics, Geosystems*, 9(10), Q10, 017.