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Application of different inverse methods for combination of v_S and v_{GPR} data to estimate porosity and water saturation

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Abstract

Inverse problem is one of the most important problems in geophysics as model parameters can be estimated from the measured data directly using inverse techniques. In this paper, applying different inverse methods on integration of S-wave and GPR velocities are investigated for estimation of porosity and water saturation. A combination of linear and nonlinear inverse problems are solved. Linear least-squares and conjugate gradient are used as linear techniques, whereas grid search and Newton methods are selected as nonlinear ones. It is understood that v_S depends on density and Lame Constant (shear modulus) and v_{GPR} on dielectric constant. This combination seems to be logical. Shear modulus is related to porosity using Bruggeman's rule. Density and dielectric constant is also related to porosity and water saturation. This implies that v_S and v_{GPR} are bivariate functions of porosity and water saturation, which are our unknown model parameters. The model parameters are estimated to minimize the cost functional ora system of the equations. In order to convert the nonlinear problem into the linear form, taking logarithm and changing variables were used. The problem was convex, which was inferred from the linear form, so there was just one local minimum as the global minimum of the problem. The grid search method shows that porosity and water saturation cannot be estimated by v_{GPR} or v_S uniquely. The results of the four methods were compared with each other and a good agreement was observed.

Keywords: GPR wave, Inverse methods, Porosity, S wave, Water saturation.

1. Introduction

The goal of inverse theory is to determine model parameters from observations (Boulanger and Chouteau, 2001). Based on the problem one or more inverse techniques can be used. In one point of view, inverse problems can be divided into two classes: continuous and discrete. However, continuous inverse problems have to be converted into discrete ones, because data are measured discretely in the field and computers can deal just with digitized inputs. There are many methods to convert continuous inverse problems into their discrete forms. The simplest method is midpoint rule which can be found in Aster et al. (2005). In another point of view which is more important, inverse problems can be classified into two main classes: linear and nonlinear inverse problems which are more complicated. There are many linear inverse methods such as classic least-squares, SVD, Tikhonov regularization and conjugate gradient. Many techniques for solving nonlinear inverse problems need to linearize the system of equations. Some techniques like taking logarithm from the equations and changing variables execute the linearization at first and then use the solving techniques related to linear inverse problems, but some other techniques like Gauss-Newton execute the linearization during their algorithms.

There are also some techniques like grid search that do not need linearization. This method is useful when the numbers of the model parameters are small and are limited to an interval like [a, b]. In this paper, minimization of a cost function is targeted such that a quantitative integration of the v_S and v_{GPR} to estimate porosity and water saturation is derived. These parameters were estimated using stochastic rock- physics modeling (Bachrach, 2006). Dannowski and

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Yaramanci used the combination of geoelectric and GPR to estimate porosity and water saturation (Dannowski and Yaramanci, 1999). In order to use this minimization, two linear and two nonlinear techniques are applied and the results are compared.

2. Methodology

At first, the problem are defined and then the interested inverse methods are introduced and applied on our problem.

2. 1. Definition of the problem

For shallow subsoil, the estimates of in-situ porosity and water saturation are important, but until now it has been difficult to estimate these reliably (Ghose and Slob, 2006). We want to obtain these parameters using the velocity integration of seismic and EM waves. Seismic S wave velocity is dependent on shear modulus and density of the medium as:

$$v_s = \sqrt{\frac{\mu}{\rho}} \tag{1}$$

where μ and ρ are bulk shear modulus and density of the medium, respectively.

GPR wave velocity is usually dependent on dielectric constant of the medium and can be calculated as:

$$v_{GPR} = \frac{c}{\sqrt{\varepsilon_b}} \tag{2}$$

where c is the speed of light in free space and ϵ_b is the dielectric constant of the medium. Lame constants are related to porosity and water saturation by using Bruggeman mixing rule (Sihvola, 1999). This can be expressed as:

$$(1-\phi)\frac{X_s - X_b}{X_s + 2X_b} + \phi S_w \frac{X_w - X_b}{X_w + 2X_b} + \phi(1-S_w)\frac{X_a - X_b}{X_a + 2X_b} = 0 \tag{3}$$

where X represents λ and μ, the subscripts b, s, w and a indicate the effective bulk property, and the properties of the constituent solid, water and air phases, respectively. The effective bulk values are obtained by solving Equation (3) with an appropriate root finding routine (Ghose and Slob, 2006). Bulk density and dielectric constant can be related to porosity and water saturation by power law:

$$Y_b^\alpha = \left(1-\phi\right) Y_s^\alpha + \phi S_w Y_w^\alpha + (1-S_w)\,\phi\, Y_a^\alpha \tag{4}$$

where Y stands for ϵ with α=0.5 for sandy soils with little clay content (Complex Refractive Index Method or CRIM; Birchak et al., 1974) and α=0.65 for clayey soils (Wang and Schmugge, 1980), and Y stands for ρ with α=1 the subscripts are same as in Equation (3). We have used the CRIM model for our case, because it provides accurate values for ϵ_b for many soil types (Ghose and Slob, 2006). Based on what was stated above, it can be said that v_S and v_{GPR} are bivariate functions of porosity and water saturation. Our integrated cost functional problem is defined as bellow:

$$c_\beta = \left(\frac{\left|V_{EM}^m(\varphi, S_w) - V_{EM}^{data}\right|^\beta}{2\left|V_{EM}^{data}\right|^\beta} + \frac{\left|V_s^m(\varphi, S_w) - V_s^{data}\right|^\beta}{2\left|V_s^{data}\right|^\beta} \right)^{1/\beta} \tag{5}$$

β=1 is for L1 norm (absolute error) and β=2 is for L2 norm (global root-mean-square error). $V_{EM}^m(\phi, S_w)$ and $V_S^m(\phi, S_w)$ are predicted velocities for GPR and seismic methods and V_{EM}^{data} and V_s^{data} are also velocity data for GPR and seismic methods, respectively. In fact, the normalized residual values are considered. The cost functional is minimized to obtain estimates for porosity and water saturation. This minimization is done using different inverse techniques in the following of this paper and the results are compared.

2. 2. Applying inverse techniques

In this section two linear and two nonlinear techniques are introduced and applied on our problem. At first, we apply nonlinear techniques because of the nonlinear nature of the problem and then the problem is linearized and finally two well-known linear inverse methods are applied.

2. 2. 1. Nonlinear inverse methods

Two nonlinear inverse methods are grid search and Newton techniques:

Grid search. One strategy for solving a nonlinear inverse problem is to exhaustively consider “every possible” solution and pick the one with the smallest error (Menke, 2012). When the trial solutions are drawn from a regular grid in model space, this procedure is called a grid search (Menke, 2012). Grid searches are most practical when

1. The total number of model parameters

is small, say M < 7. The grid is M-dimensional, so the number of trial solutions is proportional to L^M, where L is the number of trial solutions along each dimension of the grid (Menke, 2012).

2. The solution is known to lie within a specific range of values, which can be used to define the limits of the grid (Menke, 2012).

3. The cost function is smooth over the scale of the grid spacing so that the minimum is not missed through the grid spacing being too coarse (Menke, 2012).

In applying this method on the problem, the exact form of Equation (5) is chosen for β=1 and β=2.

Newton method. In order to apply Newton method on the problem, two terms in Equation (5) are considered separately for β=1 and the denominators and absolutes of the terms are not required for applying Newton method. Thus a system of equations like F(x)=0 with two equations and two model parameters exists which is very appropriate to be dealt by Newton method.

Given a system of equations F(x) = 0 and an initial solution $\mathbf{x}^0$, repeat the following steps to compute a sequence of solutions $\mathbf{x}^1$, $\mathbf{x}^2$, … . Stop when the sequence converges to a solution with F(x) = 0.

1. Use Gaussian elimination to solve

$\nabla F(x^k)\Delta x = - F(x^k)$

2. Let $x^{k+1} = x^k + \Delta x$

3. Let k=k+1.

2. 2. 2. Linear inverse methods

As it is known the problem is nonlinear, but it can be converted into a linear problem by some mathematical tricks and changes in the variables. From Equation (2) and CRIM model, the following relation can be extracted:

$$\sqrt{\varepsilon_S}(1-\phi) + \sqrt{\varepsilon_w}\phi S_w + \sqrt{\varepsilon_a}\phi(1-S_w) = \frac{c}{v_{EM}} \quad (6)$$

by assuming the following relations

$$A_1 = \sqrt{\varepsilon_W} - \sqrt{\varepsilon_b},\ B_1 = (\sqrt{\varepsilon_a} - \sqrt{\varepsilon_S}),$$
$$C_1 = c/v_{EM} - \sqrt{\varepsilon_S},\ X = \phi S_w,\ Y = \phi \quad (7)$$

Equation (6) is equivalent to the following relation:

$$A_1 X + B_1 Y = C_1 \quad (8)$$

The same procedure can be done for Equation (1) by using power law and we have:

$$(\rho_W - \rho_a)\varphi S_w + (\rho_a + 1.5\frac{\mu_S}{v_S^2} - \rho_S)\varphi = \frac{\mu_S}{v_S^2} - \rho_S \quad (9)$$

by assuming relations at below,

$$A_2 = (\rho_W - \rho_a), B_2 = (\rho_a + 1.5\frac{\mu_S}{v_S^2} - \rho_S), C_2 = (\frac{\mu_S}{v_S^2} - \rho_S) \quad (10)$$

Equation (9) is as:

$$A_2 X + B_2 Y = C_2 \quad (11)$$

Therefore, by combining the seismic and GPR velocities, a system of equations with two equations and two model parameters is derived:

$$\begin{cases} A_1 X + B_1 Y = C_1 \\ A_2 X + B_2 Y = C_2 \end{cases} \quad (12)$$

Now, the problem is a linear one and linear techniques can be utilized to solve it. Linear least-squares and conjugate gradient methods are applied on this small linear systems of equations (two equations and two model parameters). In conversion of the problem into the linear inverse problem, linearization approximation is not used. So the new linear system of equations is equivalent to the nonlinear system of equations. The data kernel matrix of the above system of equations is a positive definite matrix, so our problem is convex and any local minimum found for the inverse problem is global minimum. Consequently, it can be concluded that the solution founded by using the above techniques is the only solution of our problem.

Least-squares method: For a system of equations Gm=d the least-squares solution can be found by the following formula:

$$m^{est} = (G^T G)^{-1} G^T d \quad (13)$$

The only required condition for using this classic method is that G matrix be full column rank matrix. In our system of equations G is full rank matrix, thus, the problem has the necessary condition for the least-squares technique.

Conjugate gradient (CG) method: conjugate gradient method is an iterative inverse technique to solve a symmetric and positive definite system of equations that its algorithm does not need to calculate the inverse matrix of G. Like every iterative inverse method, conjugate gradient has special application for problems with large matrices. However, the matrix of our problem is very small.

Conjugate gradient algorithm for a positive definite and symmetric system of equations $Ax = b$, and an initial solution x_0, $p_0=r_0=b-Ax_0$. Repeat the following steps until convergence occurred.

1. Let $\alpha_k=(r_k^T r_k) / (p_k A p_k)$.
2. Let $x_{k+1}=x_k+\alpha_k p_k$.
3. Let $r_{k+1}=r_k-\alpha_k A p_k$.
4. Let $\beta_k=(r_{k+1}^T r_{k+1}) / (r_k^T r_k)$.
5. Let $p_{k+1}=r_k+\alpha_k p_k$.

A major advantage of the CG method is that it requires storage only for the vectors x_k, p_k, r_k and the matrix A (Aster et al., 2005). In theory, the algorithm finds an exact solution to the system of equations in n iterations -A is an n by n matrix for the algorithm (Aster et al., 2005). In practice, because of round-off errors in the computation, the exact solution may not be obtained in *n* iterations (Aster et al., 2005).

3. The numerical results model

The model consists of two layers: sand and shale. The focus is on the second layer and by knowing the v_S and v_{GPR} of this layer, the calculation of the model parameters (porosity and water saturation) using different inverse methods. However, the same work can be done for the first layer and its model parameters can also be estimated. In the following, numerical results related to applying different inverse method are represented.

At first, each of the two terms in Equation (5) are considered exclusively and it can be observed that none of them can give the porosity and water saturation of the medium solitary, but combining the two terms give us model parameters uniquely. All of the related results are in figures 1 to 4 (color bars are in terms of dB). The estimated porosity and water saturation for β=1 are 33% and 60% and for β=2 are 33% and 61%, respectively. Therefore, the difference between the results of the norms is trivial and they are in good agreement.

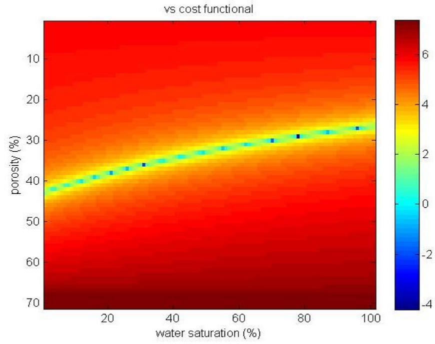

Fig. 1. Cost functional of v_s. The green line is indicative of local minima. Model parameters can’t be estimated uniquely by v_s cost functional.

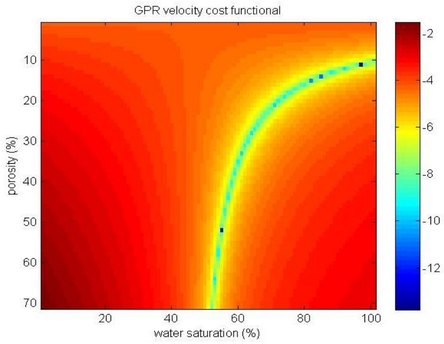

Fig. 2. Cost functional of v_{GPR}. The green line is indicative of local minima. Model parameters can’t be estimated uniquely by v_{GPR} cost functional.

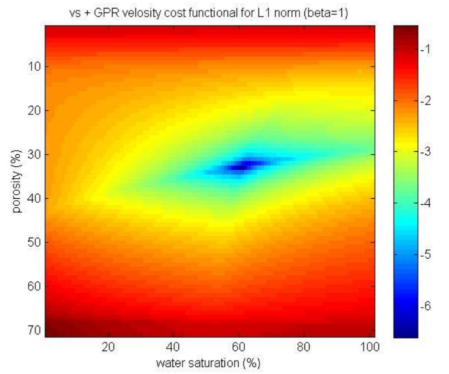

Fig. 3. Cost functional of $v_s + v_{GPR}$ for β=1. In contrast to Figs. 1 and 2, model parameters can be estimated uniquely.

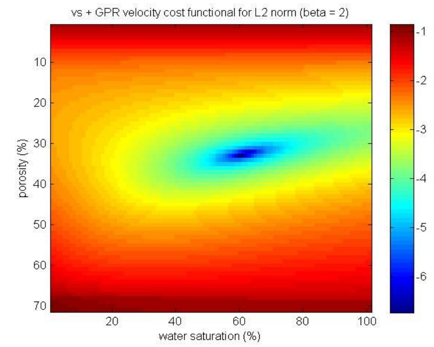

Fig. 4. Cost functional of $v_s + v_{GPR}$ for β=2. In contrast to Figs. 1 and 2, model parameters can be estimated uniquely. The results for model parameters are in good agreement with Fig. 3.

Table 1. Layer parameters

	Vs (m/s)	V_{EM} (m/s)	ρ (g/cc)	ϕ (%)	S_w (%)	σ (s/m)
Layer 1 sand	**1500**	**5.7×10⁷**	**1.40**	**40**	**60**	**0**
Layer 2 shale	**1400**	**6.1×10⁷**	**1.18**	**30**	**65**	**0**

Table 2. Converging trend of the Newton algorithm to the unique solution of the problem

	1	2	3	4	5	6	7	8	9
Φ = x(1)	**0.3551**	**0.3384**	**0.3302**	**0.3050**	**0.3004**	**0.3003**	**0.3003**	**0.3003**	**0.3003**
S_w = x(2)	**1.7008**	**-0.0755**	**0.3344**	**0.5512**	**0.6089**	**0.6115**	**0.6115**	**0.6115**	**0.6115**

Table 3. Converging trend of the CG algorithm to the unique solution of the problem

	0	1	2	3	4
X=Φ*S_w	**0**	**0.1883**	**0.1828**	**0.1828**	**0.1828**
Y=Φ	**0**	**0.2973**	**0.3008**	**0.3008**	**0.3008**

Newton method. The MATLAB code that is written for Newton algorithm needs to run in six iterations to be converged. The estimated model parameters are 30% for porosity and 61% for water saturation. Initial solution chosen to start the algorithm is 10% for both model parameters, means $x^0 = [0.1; 0.1]$. The converging trend of the algorithm is shown in Table 2.

Least-squares method. If one substitute the corresponding values for the known parameters in Equation (13), the following linear system of equations is derived:

$$\begin{cases} 8X - 3.5Y = 0.41 \\ 998.7X + 1576.4Y = 656.7 \end{cases} \tag{14}$$

Using relation (6), the obtained estimates for porosity and water saturation are 30% and 61%, respectively. They are same as the estimated values from Newton technique.

Conjugate gradient (CG) method, this method is applied on Equation (14) and the converging trend of the algorithm to the unique solution of the problem is shown in Table 3. Estimated values for porosity and water saturation are 30% and 61%, respectively, which are same as the two previous methods (Newton and LS methods) and have little difference with grid search method for both norms. As can be observed from Table 3, the algorithm reaches the convergence in two iterations (hvaing two model parameters), which is not seen usually in practice. Here it is occurred, because number of the model parameters and equations is very small, consequently round-off errors are very small to make slow the converging trend of the algorithm. The second column of the Table 2 is the initial solution for algorithm.

4. Discussions

Two subjects can be discussed here: 1. combination of the v_s and v_{GPR} to estimate porosity and water saturation uniquely, 2. importance of linearization of the nonlinear inverse problems.

4. 1. Combination of the v_s and v_{GPR} to estimate porosity and water saturation uniquely

It should be noted that seismic and GPR methods have different criteria to distinguish earth layers. Thus, the studied model in this paper is an ideal model and it can’t be seen in practice. For instance, the first layer in seismic is in depth of 5 (m) but for GPR depth of the first layer is in 2 (m), therefore, the layer bounds do not correspond with each other. This is the most important problem that challenge the method for practical applications. The only solution for this problem is to make a joint model so that the boundary of the layers of this joint model are in an increasing trend. For the previous example, the boundaries of the joint model are in 2 and 5 (m) respectively. This is a theoretical solution for the problem might works in practice. Another important problem is to select the values of Lame constants and dielectric constant for solid part of the medium, because the values in tables are for solids in their ideal states and these ideal states cannot be found in real world and, therefore, this affects our results. If the method works well in practice for 1-D estimation of porosity and water saturation, it will work for 2-D and 3-D cases.

4. 2. Importance of linearization of the nonlinear inverse problems

One of the advantages of linearization is that applying linear inverse methods is easier than nonlinear techniques. In this paper advantage of the linearization has been shown. Conversion of the problem into a linear system of equation as Ax=b indicated that the A matrix was a positive definite matrix and concluded that the problem has just one local minimum which is also global minimum. This matter can’t be extracted from the system of equations in nonlinear form.

It can be asserted that the best conversion of nonlinear inverse problem to linear one is done by taking logarithm and changing variables, because linearization approximations are not used and the corresponding linear form is equivalent with nonlinear inverse problem. However, these techniques cannot be done for all of the nonlinear inverse problems.

5. Conclusions

For shallow subsoil, the estimates of in-situ porosity and water saturation are important. Both of them can be estimated uniquely by combining velocities of seismic and ground penetrating radar methods. This combination and finding porosity and water saturation (model parameters) produces a nonlinear inverse problem. A two layered model (first layer: sand, second layer: shale) was used and our focus was on the second layer. Here these model parameters were estimated by different inverse methods. Two different works were done in this paper: 1) estimation of porosity and water saturation by combining seismic and GPR methods, 2) applying different inverse methods on the problem and comparing the results as the main purpose of this paper. Two linear and two nonlinear inverse techniques were used. To convert the problem into linear form, taking logarithm and changing variables were employed. The problem was convex, which was inferred from the linear form, so there was just one local minimum as the global minimum of the problem. Classic least squares and conjugate gradient were used as linear methods and Newton and grid search were applied as nonlinear techniques. The results were compared with each other and they show a very good agreement. The most challenging matter for applying the method in practice is that the layer bounds in seismic and GPR methods does not correspond with each other. It can be concluded that conversion of the problem into linear one and solving it is very better than solving it in its nonlinear form.

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