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(Research Article)

Effect of Nonextensive Perturbation on Ion Acoustic Solitons

Pakzad, H. R.[1]

1. **Corresponding Author**, Department of Physics, Bojnourd Branch, Islamic Azad University, Bojnourd, Iran. E-mail: pakzad@bojnourdiau.ac.ir

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Abstract
The behavior of ion acoustic wave (IAW) is studied in an electron-ion plasma consisting of cold ions and nonextensive electrons. In this study, the reductive perturbation method is used and the new point is the non-uniformity of the nonextensive parameter in the media. We want to achieve more realistic results of ion acoustic wave behavior by better using the reductive method. In fact, the variation in the behavior of ion acoustic wave when it encounters the nonextensity perturbation region is examined. Perturbation area is a part of plasma where the nonextensivity changes slightly. Therefore, the presence of nonextensivity is introduced as the first order perturbation and the phase velocity is applied as a fixed parameter in the calculations. The modified KdV (mKdV) equation is derived to describe the behavior of the ion acoustic wave propagation in this model. The obtained equation clarifies the change of the soliton profile when moving in all through the perturbation area. Our numerical results show that part of ion acoustic waves propagates as oscillatory shock wave in the perturbed area. The results of this investigation can be helpful for understanding the behavior of ion acoustic waves in an astrophysical environment and space plasmas with varying nonextensivity (Qiu *et al.*, 2020; Silva et al., 1998; Lima *et al.*, 2000).

Keywords: Ion acoustic; Soliton; Shock; Modified KdV equation; Perturbation; Nonextensivity.

1. Introduction
One of the interesting branches in plasma physics is the study of linear and nonlinear propagation of ion acoustic waves (Ikezi et al., 1970; Schamel, 1980; Longren, 1983; Nakamura et al., 1993; Barkan et al., 1996; Kourakis & Shukla, 2003; Lee, 2009; Dubinov & Kolotkov, 2012; Verheest et al., 2013; Sultana, 2018; Dubinov et al., 2021; Liu et al., 2018). There is a variety of reports about non-uniform plasmas and many authors have studied them in different ways (Asano, 1974; John & Saxena 1976; Gell & Gomberoff, 1977; Rao & Varma, 1977; Goswami & Sinha, 1976; Talbot et al., 1980; Shukla & Mamun, 2002; Cousens et al., 2012; Chaudhuri et al., 2019). Talbot et al. (1980) provided an expression for the thermophoretic force by considering the non-uniformity in the velocity distribution of a neutral gas. Shukla & Mamun (2002) considered the non-uniformity of the plasma as the density gradient of unperturbed number densities and studied the dispersion properties of electrostatic and electromagnetic waves. One of the theoretical methods to study the behavior of ion acoustic waves in plasmas is the reductive perturbation method (Washimi & Taniuti, 1966). In this method, independent variable $\xi = \varepsilon(x - \lambda\tau)$ is used in which the phase velocity (λ) is assumed constant. Although the results of many studies show that λ is dependent on the plasma parameters (Schamel, 1973; Roy & Sahu, 2020; Singh et al., 2018; Maksimovic et al. 1997; Goswami et al., 2020; Pakzad & Tribeche, 2013; Hafez, 2019), the temperature, density and energy distribution of particles are not fixed. Therefore, the previous results are valid as long as the defined parameters (in phase velocity) are uniform in the environment. Recently, in an interesting work, Chaudhuri et al. (2019) used the perturbation expansion of temperature explained by Cousens et al. (2012) in their non-uniform model. For the mentioned reasons, we are going to make corrections in how to use the reductive perturbation method and then we define the non-extensive parameter of electrons as a first-order perturbation. In this study, considering the nonextensive parameter as non-uniform (Qiu et al., 2020; Silva et al., 1998; Lima et al., 2000) and introducing it as a first-order expansion in reduced

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perturbation calculations, is a new idea and innovation for a better and more realistic understanding of how wave propagates in a q-plasma.

2. Method

We use the nonlinear dynamics of ion acoustic wave propagation for our considered model (Lu & Liu, 2021).

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=\circ \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n \tag{3}$$

where, n is the ion number density normalized by its equilibrium value n_{0i}, u is the ion fluid speed normalized by c_i, and ϕ is the electrostatic wave potential normalized by T_e/e where T_e is the electron temperature. The time t and the distance x are normalized by the ion plasma frequency $\omega_{pi}^{-1}=\sqrt{\frac{m_i}{4\pi n_{0i}e^2}}$ where m_i is ion mass and the Debye radius $\lambda_{Di}=\sqrt{\frac{T_e}{4\pi n_{i0}e^2}}$, respectively.

Over the last three decades, many studies have been done in nonextensive statistic mechanics field based on the deviations of the Boltzmann–Gibbs–Shannon (BGS) entropic measure. A suitable nonextensive generalization of the BGS entropy for statistical equilibrium was presented by Tsallis (1988), suitably extending the standard additivity of the entropies to the nonextensive case where the entropic index q underpins the generalized entropy of Tsallis and measures the amount of its nonextensivity of the system. In this case, normalized distribution of electrons is given by (Rehman & Lee, 2018):

$$n_e=[1+(q-1)\phi]^{\frac{q+1}{2(q-1)}} \tag{4}$$

In statistical mechanics and thermodynamics, systems characterized by the property of nonextensivity are systems for which the entropy of the whole is different from the sum of the entropies of the respective parts. In other words, the generalized entropy of the whole is greater than the sum of the entropies of the parts if $q<1$ (superextensivity), whereas the generalized entropy of the system is smaller than the sum of the entropies of the parts if $q>1$ (subextensivity). Moreover, $q=1$ corresponds to the standard, extensive, BGS statistics. Nonextensive statistics were successfully applied to a number of astrophysical and cosmological scenarios, which include stellar polytropes (Plastino & Plastino, 1993), the solar neutrino problem (Kaniadakis et al., 1996), peculiar velocity distributions of galaxies (Lavagno et al., 1998) and generally systems with long-range interactions and fractals such as space-times. Cosmological implications were discussed in Torres et al. (1997), and recently an analysis of plasma oscillations in a collisionless thermal plasma was provided from q-statistics in Lima et al. (2000).

Many problems about non-uniform plasmas have been treated by the reduced perturbation method (Gell & Gomberoff, 1977; Rao & Varma, 1979; Goswami & Sinha, 1976; Havnes et al. 2001; Cousens et al., 2012; Chaudhuri et al., 2019). In order to use this method, we introduce the independent variables through the stretched coordinates of the weakly nonlinear theory of the electrostatic waves with small but finite amplitude as follows:

$$\xi=\varepsilon^{\frac{1}{2}}(x-\lambda\tau),\ \tau=\varepsilon^{\frac{3}{2}}t \tag{5}$$

$$n=1+\varepsilon n_1+\varepsilon^2 n_2+ \tag{6}$$

$$u=\varepsilon u_1+\varepsilon^2 u_2+\cdots \tag{7}$$

$$\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots.. \tag{8}$$

and the dependent variables are expanded as: where ε is a small dimensionless parameter measuring the weakness of the dispersion and nonlinearity, and λ is the phase velocity normalized by the linear IA speed (C_i).

Let us not forget that our main problem is the non-uniformity of the non-extensive parameter in the environment. In this manuscript, the term "non-uniform" is used to mean "spatially inhomogeneous", and thus in the reductive perturbation method, perturbed parameter should be determined by the order of the non-uniformity (Chaudhuri et al., 2019). In this situation, we introduced a weak perturbation of the q-parameter and present it as a small but finite $\mathrm{q}=\mathrm{q}_0+\varepsilon^1\mathrm{q}_1$, where q_0 is related to the uniform part of the nonextensivity, in most part of the plasma, and q_1 is the first order perturbation of it in a limited area of the environment. For example, for $q_0 = 1$, the plasma is in the thermal state and for the perturbed area of this parameter, nonextensivity appears as q_1. In fact, with this choice, the non-uniformity of the non-extensive effect on ion acoustic wave propagation is weakly assumed.

Now we use the stretched coordinates and expansions (5-7) in the basic normalized equation (1-3), and collect the same terms in different powers of ε. The final result is the modified Korteweg-de Vries (mKdV) equation.

$$\frac{\partial\phi_1}{\partial\tau} + (\frac{3}{2\lambda} - \frac{3-q_0}{4}\lambda)\phi_1\frac{\partial\phi_1}{\partial\xi} + \frac{\lambda^3}{2}\frac{\partial^3\phi_1}{\partial\xi^3} + \frac{\lambda^3}{4}\frac{\partial(q_1\phi_1)}{\partial\xi} = 0 \quad (9)$$

where $\lambda = \sqrt{\frac{2}{1+q_0}}$.

It is seen that the reductive perturbation theory is extended by considering the perturbation in the nonextensivity, and its result appeared in Equation (9). Perturbation in the q appears in the last term of the above equation and the constant phase velocity is also guaranteed (Landau does not depend on the variation of q_1).

3. Measurements

It can be seen that λ is independent of the q_1 parameter. Equation (9) describes the propagation of small amplitude ion acoustic wave in the presence of nonextensive perturbation parameter within the plasma. The last term in this equation presents a new source of dissipation, which depends on the appearance of perturbation in the electron distribution in plasma. This term demonstrates that the weak perturbation of the nonextensivity (q_1) acts as a source of dissipation. In fact, Equation (9) explains the evolution of IA solitary waves in collision with an area with different nonextensivity, which was introduced before. In the absence of perturbation effect (q_1=0), Korteweg-de Vries (KdV) equation describes the solitary waves in uniform nonextensive plasma (Bacha et al., 2012):

$$\frac{\partial\phi_1}{\partial\tau} + \left(\frac{3}{2\lambda} - \frac{3-q_0}{4}\lambda\right)\phi_1\frac{\partial\phi_1}{\partial\xi} + \frac{\lambda^3}{2}\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \quad (10)$$

which admits a solitary wave solution as follows $\phi_1 = \phi_0 sec\,h^2(\frac{\chi}{w})$ where $\chi = \xi - u\tau$, $\phi_0 = 3u/A$ is the soliton amplitude and $w = 2\sqrt{B/u}$ is its width, where $A = (\frac{3}{2\lambda} - \frac{3-q_0}{4}\lambda)$ and $B = \frac{\lambda^3}{2}$. It is emphasized that the initial solitary solution is defined out of the perturbation region.

4. Results and Discussion

Studying the behavior of the ion acoustic solitary wave after interaction with the q-perturbation space can be interesting. It can be expected that due to the presence of the fourth term ($\frac{\lambda^3}{4}\frac{\partial(q_1\phi_1)}{\partial\xi}$) in Equation (9), the initial solitary waves change in the perturbed part of the plasma. It is reminded that the "non-uniform" is used in the concept of "spatially inhomogeneous". For this purpose, we consider a step perturbation for nonextensivity as:

$$q_1 = q_{01}[1 + \tanh(\alpha\xi)] \quad (11)$$

The above relation indicates that the nonextensivity of electrons reaches to a maximum value ($2q_{01}$) at the perturbation region and it gives a zero value far enough from $\xi = 0$ at $\rightarrow -\infty$. We can control the thickness of varying area through the parameter α. Accordingly, a schematic of the slope and slow variation of q_1 is shown in Figure 1.

We can consider q_1 as a constant value in Equation (9) (as assumed in the following). However, according to Figure 1, we assume that the change in the energy perturbation of the particles appears slowly and stepwise (we could also consider the perturbation effect as Gaussian).

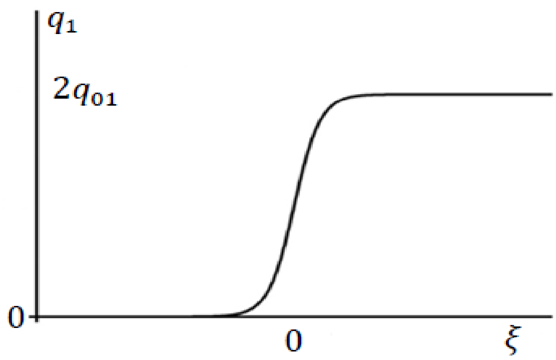

Figure 1. Perturbation of nonextensivity (q_1) as a function of ξ.

Let us review once again the important point and the main challenge raised in this study. We have pointed out that the phase velocity in $\xi = \varepsilon^{\frac{1}{2}}(x - \lambda\tau)$ is constant and depends on the plasma parameters. Therefore, the study of solitonic solutions of the KdV equation in a plasma model with phase velocity and therefore with constant parameters of plasma will be allowed unless we compare different plasmas. To improve this problem, we used a weak perturbed parameter ($q = q_0 + \varepsilon^1 q_1$) to get as close as possible to the actual model. We use numerical calculations to investigate the changes in the initial solution of the ion acoustic wave (soliton) in passing through the area that has excess/deficiency nonextensivity. In this case, we use the fourth-order Runge-Kutta method to solve Equation (9). The place grid spacing is selected $\Delta\xi$=0.001 and 0.005 (as cross check for the numerical stability of solution) and the time grid spacing is chosen as $\Delta\tau$= 0.0001. We examine the effect of q_0, q_{01} and α parameters on the ion acoustic solitary waves when it passes through the perturbation area. Therefore, from Figures 2 to 4, it is quite obvious that the ion acoustic waves radiate some of their energy as backward moving oscillatory shock wave after passing through the perturbation region ($q_1 = q_{01}[1 + \tanh(\alpha\xi)]$). In Figure 2, the initial ion acoustic soliton travels with speed u=0.3 in the unperturbed region (ξ<0) and then interacts with the nonextensive perturbation q_1=0.15(1+tanh0.8ξ) in $\xi \geq 0$, and is finally imaged in the ξ>0 region. In this Figure, time evolution of soliton is shown at τ=3, τ=5 and τ=8 in ξ>0. It is observed that in the presence of excess q-parameter in part of the environment, ion acoustic soliton propagation is accompanied by shock wave oscillatory. It is clear that the amplitude of the wave increases and also the oscillating wave occurs at the point where the initial soliton wave collides with the perturbation of nonextensivity. Figure 3 shows the time evolution of the soliton wave after entering the negative perturbation of q-parameter (q_1=-0.15(1+tanh0.8ξ)) in the plasma at three different times. It is observed that the amplitude of the wave decreases after entering the perturbation area, but a strong shock wave is generated. Figure 3 illustrates that when ion acoustic soliton in a positive nonextensive plasma experiences a negative perturbation of nonextensivity in part of the media, it loses much of its energy. A comparison of Figures 2 and 3 shows that the negative perturbation effect ($q_1 < 0$) is stronger on the initial soliton.

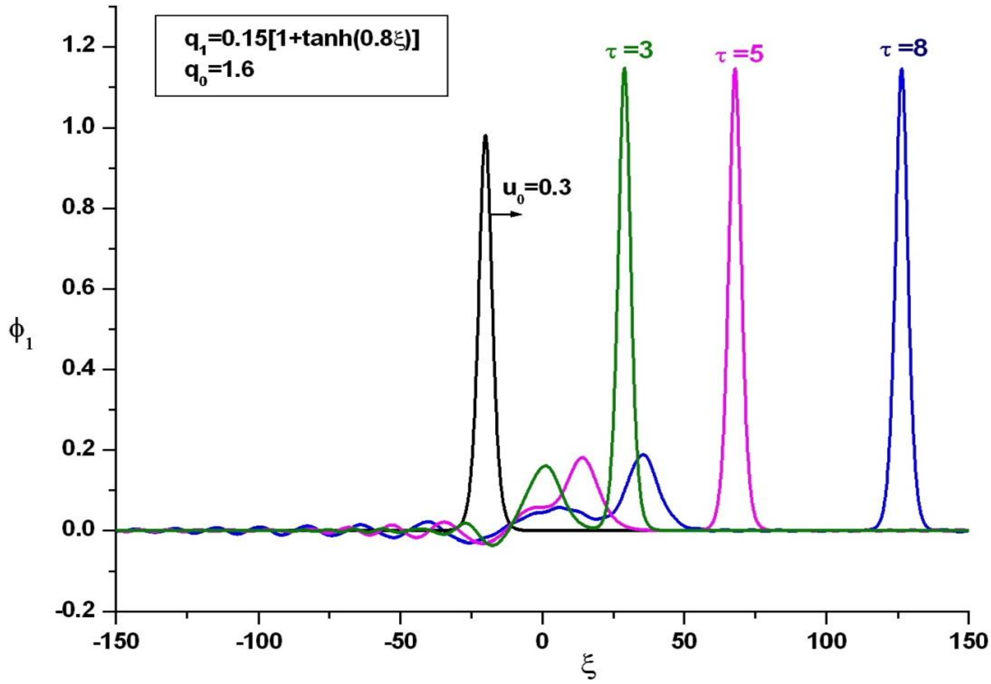

Figure 2. Time evolution of the ion acoustic soliton in dealing with a positive nonextensive perturbation region.

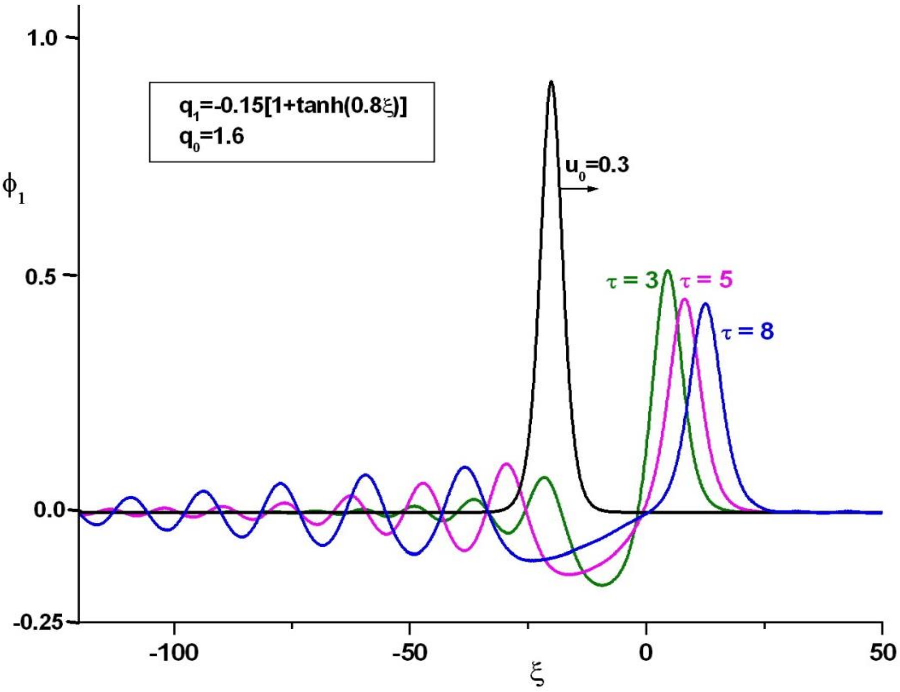

Figure 3. Time evolution of the ion acoustic soliton in dealing with a negative nonextensive perturbation region.

In Figure 4, we compare the effect of two positive and negative perturbations ($q_1 = -0.2$ and $q_1 = +0.2$) on the ion acoustic soliton when $q_0 = -0.6$. In this case, due to the constant assumption of the perturbed parameter (q_1), the fourth term of Equation (9) can be presented as $\frac{\lambda^3}{4} q_1 \frac{\partial \phi_1}{\partial \xi}$. This figure shows that there is no change in the initial soliton in the passing of negative perturbation ($q_1 = -0.2$), in contrast to the positive perturbation ($q_1 = +0.2$) it strongly amplifies the waves and shock oscillations created in it.

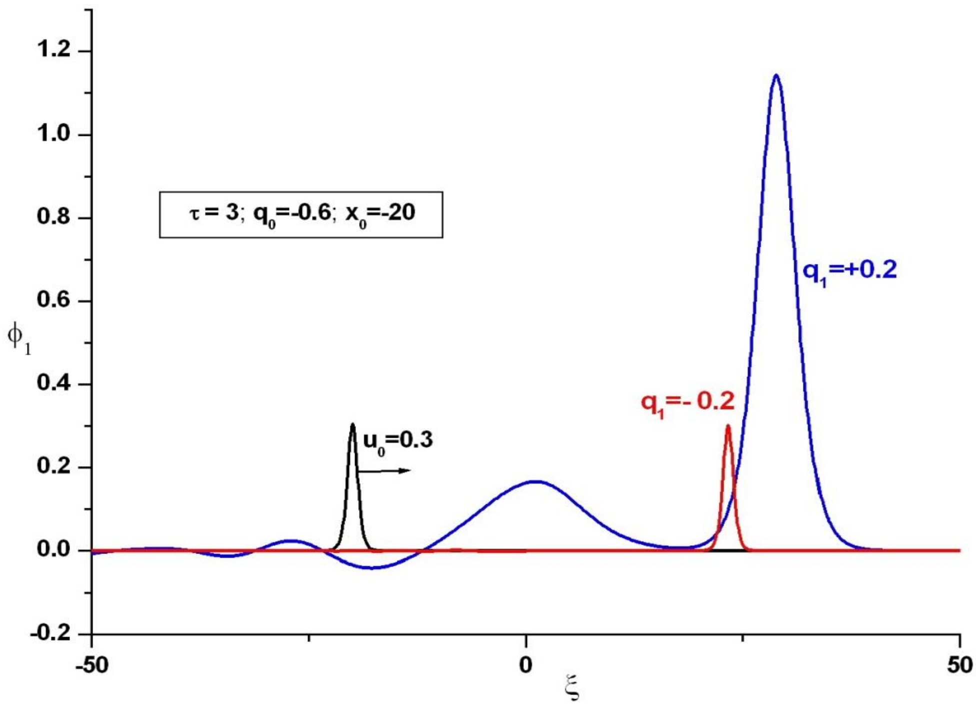

Figure 4. Time evolution of the ion acoustic soliton in dealing with a negative ($q_1 = -0.2$) and positive ($q_1 = +0.2$) nonextensive perturbation region for $q_0 = -0.6$.

One of the most important and interesting results of this study can be seen in Figures 5 and 6. The soliton wave is moving from ξ=-20 with speed $u_0 = 0.3$ towards positive perturbations $(q_1 = 0.1,\ 0.3 \text{ and } 0.5)$ in Figure 5 and negative perturbations $(q_1 = -0.1, -0.3 \text{ and } -0.5)$ in Figure 6. The background value of nonextensivity (q_0) has been considered equal to 1.6. It is seen that the wave propagates with greater speed and amplitude in the presence of positive perturbation. In contrast, it is clear from Figure 6 that if the wave encounters a negative type of perturbation, it weakens so that with increasing intensity of the perturbation, the speed and intensity of the wave extinction get stronger. Figure 7 shows that the effect of a non-extensive perturbation is negligible for a wave traveling at high speed $(u_0 = 0.5)$. As can be seen, the amount of perturbation has no effect on this result. In other words, if the velocity of ion acoustic wave is high, the non-extensive perturbation has no effect on the wave propagation. This outcome holds for other values (q_0, q_1 and u_0) as well.

It is depicted that the first-order perturbation of nonextensivity provides a new term in the motion equation of solitary wave, which depends on the spatial variation of perturbation as well as original profile of propagating localized wave. Furthermore, by using the numerical calculation, it is obvious that the velocity and amplitude of out coming ion acoustic wave from perturbation area is different from the primary one.

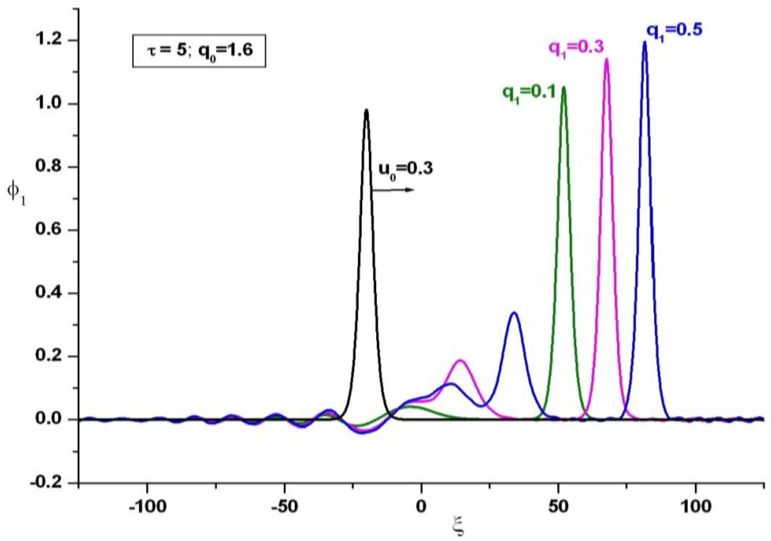

Figure 5. Time evolution of the ion acoustic soliton in dealing with three positive nonextensive perturbation regions $(q_1 = 0.1, 0.3 \text{ and } 0.5)$ for $q_0 = 1.6$.

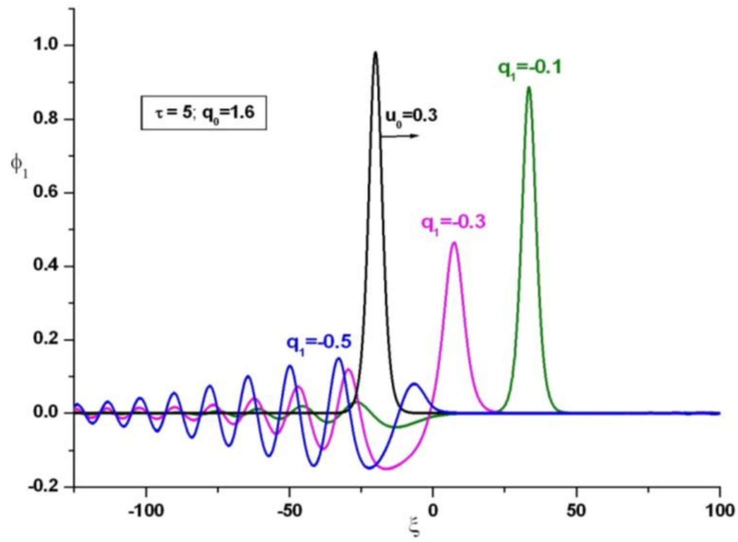

Figure 6. Time evolution of the ion acoustic soliton in dealing with three negative nonextensive perturbation regions $(q_1 = -0.1, -0.3 \text{ and } -0.5)$ for $q_0 = 1.6$.

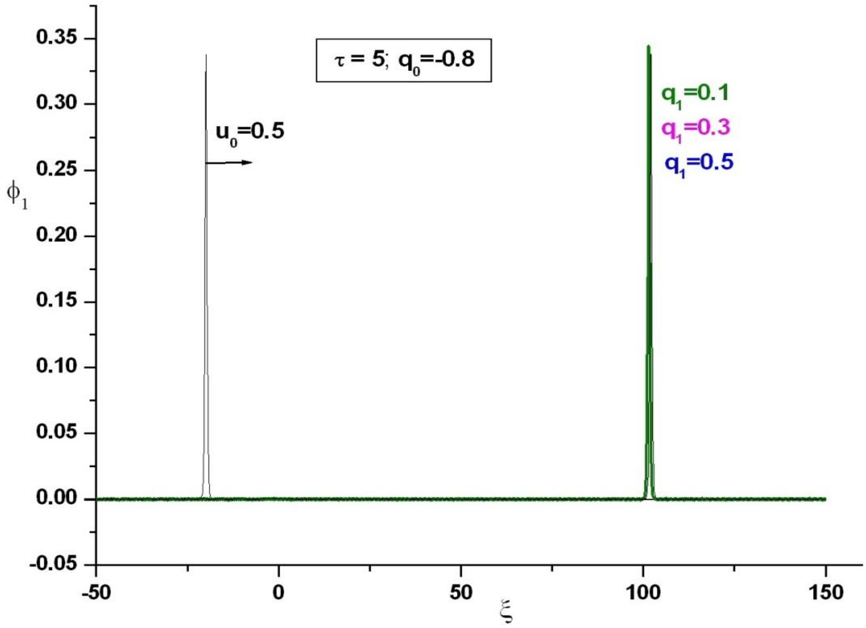

Figure 7. Time evolution of the ion acoustic soliton in dealing with three negative nonextensive perturbation regions ($q_1 = 0.1, 0.3$ and 0.5) for $q_0 = 1.6$.

5. Conclusions

We considered an electron-ion plasma with non-uniform nonextensive electrons. The reductive perturbation technique is employed to describe the propagation of ion acoustic solitary wave. We have presented a safe way to include perturbation due to temperature changes in the evolution equation of the solitary wave. In our introduced method, the phase velocity depends on the plasma parameters ($\lambda = \sqrt{2/1 + q_0}$) and remains constant throughout the environment. q_0 is unperturbed parameter of nonextensive electrons distribution. However in real conditions, the energy distribution of electrons is not uniform and therefore to improve this problem we entered the nonextensive parameter as a first-order perturbation as $q = q_0 + \epsilon^1 q_1$ in the calculations in which q_1 is perturbed parameter in a part of the distribution function. It is shown that the first-order perturbation provides a new term ($\frac{\lambda^3}{4}\frac{\partial(q_1\phi_1)}{\partial\xi}$) in the solitary wave equation of motion, which depends on the spatial variation of perturbation as well as original profile of propagating localized wave. We studied the time evolution of ion acoustic solitons during propagation in the step-shaped perturbed region of the non-extensive parameter. Using the numerical calculation, it is depicted that the amplitude of out coming ion acoustic wave from positive (negative) perturbation area is longer (shorter) than primary one. As a final result of this study, it can be claimed that any small change in the q parameter can be made a challenge to the soliton profile in the environment unless the soliton wave velocity is high enough for passing from the perturbation area. It can be said that any parameter such as temperature, density, superthermality and viscosity can be introduced as sources of oscillating shock wave production in the environment. This manuscript is important from the point of view that we know the plasma parameters that are not uniform in a realistic situation.

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