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Orbit integration in non-inertial frames

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Abstract

A precise orbit of a low Earth orbiting satellite helps us to compute the long wavelength structure part of the gravity field of the Earth. There are different methods and frames for orbit integration depending on the problem and satellite mission.

In this paper, the dynamic equations of the satellite motion are presented in different frames of navigation. A simple numerical study on a satellite orbit in local frames is also included. In these frames the geodetic coordinate of the satellite is directly integrated. Numerical studies confirm that the north-east-down frame is not stable for orbit integration either. However, the paper shows how to solve this problem by choosing a wander frame and its ability.

Key words: Frame, Orbit, Integration, Potential, Gravity

محاسبه مدار در چارچوب‌های نالَخت

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چکیده

مدار دقیق ماهواره‌های نزدیک سطح زمین در مطالعه ساختار بلند میدان جاذبه زمین بسیار مفیدند. روش‌ها و چارچوب‌های گوناگون برای محاسبه چنین مدارهای موجود هست که با توجه به مسئله مورد نظر و ماموریت ماهواره انتخاب می‌شوند. در این مقاله معادلات حرکت یک ماهواره در چارچوب‌های مختلف ارائه می‌شوند. محاسبات عددی بیانگر این مطلب هست که چارچوب‌های محلی متمایل به شمال زمین مناسب برای محاسبه مدار نیستند ولی استفاده از چارچوب معلق که حالت خاصی از چارچوب محلی هست امکان حل مدار را در چنین چارچوب‌هائی نیز حاصل می‌نماید.

چارچوب‌های انتگرال‌گیری مدار عبارت‌اند از : چارچوب لَخت، چارچوب بیضوئی، محلی و معلق. چارچوب لخت یکی از ساده‌ترین چارچوب‌های انتگرال‌گیری است و به تناوب از سوی محققین گوناگون در زمینه ناوبری و ژئودزی فضائی مورد استفاده قرار می‌گیرد. چارچوب بیضوی بسیار مشابه به چارچوب لخت است ولی به‌جای انتگرال‌گیری نسبت به مختصات دکارتی ماهواره، محاسبات مستقیما روی عرض، طول و ارتفاع ژئودتیکی صورت می‌گیرد. چارچوب‌های محلی بیشتر مناسب برای ناوبری هواپیما و روش‌های هوابرد گرانی‌سنجی‌اند و به علت همگرائی نصف‌النهارها در مناطق نزدیک قطب انتگرال‌گیری از شتاب در باند ارتفاع دچار ناپایداری می‌شود و چنانچه طول مدت پرواز طولانی باشد، ناپایداری از این باند روی محاسبه عرض و طول ژئودتیکی نیز تاثیر می‌گذارد و در نتیجه آنها را نیز ناپایدار می‌سازد. یکی از روش‌های حل این ناپایداری استفاده از چارچوب معلق است که به چارچوب محلی شباهت دارد ولی وجود یک آزیموت معلق جهت مقابله با همگرائی نصف‌النهارها ناپایداری را برطرف می‌کند. به عبارت دیگر چارچوب معلق انعطاف‌پذیرتر از محلی است. مبحث ناوبری ماهواره‌ای کاملاً مشابه با ناوبری هوائی است، با این تفاوت که ماهواره به تناوب از نزدیکی قطب‌ها می‌گذرد و درگیر با تقارب نصف‌النهارها است و در هر گردش به دور زمین، به ناپایداری در باند ارتفاعی دچار می‌شود و چون ماموریت ماهواره‌ها طولانی است، قطعاً این ناپایداری بر موقعیت مسطحاتی ماهواره نیز مؤثر خواهد بود. همان‌طور ذکر شد یکی از روش‌های حل این ناپایداری، استفاده از چارچوب معلق است. در این مقاله ما معادلات حرکت ماهواره را به این چارچوب گسترش داده و به شکل زیر ساده‌سازی کرده‌ایم:

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$$\frac{d}{dt}\begin{bmatrix} v_1^w \\ v_2^w \\ v_3^w \\ \phi \\ \lambda \\ h \\ \alpha \end{bmatrix} = \begin{bmatrix} a_1^w + 2\omega_e(v_3^w \sin\alpha\cos\phi - v_2^w \sin\phi) + \dfrac{v_3^w v_1^w}{(N+h)} - \dfrac{Me'^2 \cos^2\phi\cos\alpha(v_2^w \sin\alpha - v_1^w \cos\alpha)}{(N+h)(M+h)} + \bar{g}_1^w \\ a_2^w + 2\omega_e(v_1^w \sin\phi + v_3^w \cos\alpha\cos\phi) + \dfrac{v_3^w v_2^w}{(N+h)} - \dfrac{Me'^2 \cos^2\phi\sin\alpha(v_2^w \sin\alpha + v_1^w \cos\alpha)}{(N+h)(M+h)} + \bar{g}_2^w \\ a_3^w - \dfrac{(v_1^w)^2 + (v_1^w)^2}{(N+h)} - \dfrac{Me'^2 \cos^2\phi\left((v_1^w)^2 \cos^2\alpha - (v_2^w)^2 \sin^2\alpha + 2v_1^w v_2^w \sin\alpha\cos\alpha\right)}{(N+h)(M+h)} + \bar{g}_3^w \\ (v_1^w \cos\alpha - v_2^w \sin\alpha)/(M+h) \\ (v_1^w \sin\alpha + v_2^w \cos\alpha)/(N+h)\cos\phi \\ -v_3^w \\ -\dfrac{(v_1^w \sin\alpha + v_2^w \cos\alpha)}{(N+h)}\tan\phi \end{bmatrix}$$

که در این رابطه $M = a\left(1-e^2\right) / \left(1-e^2 \sin^2\phi\right)^{\frac{3}{2}}$ شعاع انحنای نصف‌النهاری و $N = a / \left(1-e^2 \sin^2\phi\right)^{\frac{1}{2}}$ شعاع انحنای دایره قائم اولیه در نقطه‌ای با عرض و ارتفاع ژئودتیک h و φ است. λ عرض ژئودتیک، e^2 اولین خروج از مرکز، e'^2 دومین خروج از مرکز بیضوی مرجع، α آزیموت معلق و v^w, v_2^w و v_3^w مؤلفه‌های بردار سرعت، $\bar{g}_1^w$، $\bar{g}_2^w$ و $\bar{g}_3^w$ مؤلفه‌های بردار گرانش، a_1^w، a_2^w a_3^w مؤلفه‌های بردار شتاب در چارچوب معلق است.

شکل ۱ مدار یک ماهواره نزدیک سطح زمین در یک دور گردش به دور زمین را در چارچوب معلق نمایش می‌دهد. همان‌طور که در شکل مشاهده می‌شود هیچ‌گونه ناپایداری در باند ارتفاعی و همچنین سرعت ارتفاعی وجود ندارد که بیانگر صحت محاسبه‌ها و کارآمد بودن چارچوب معلق در حل مدار ماهواره است. همان‌طور که شکل نشان می‌دهد آزیموت معلق، برای جبران تقارب نصف‌النهارها تا ۱۶۰ درجه تغییر می‌کند.

واژه‌های کلیدی: چارچوب، مدار، انتگرال‌گیری، پتانسیل، ثقل

1 INTRODUCTION

There are different frames for navigation, such as the inertial frame, ellipsoidal frame, north-east-down (NED) or navigation frame, and wander azimuth frame or wander frame. Differential equations of motion can be solved in each one of the mentioned frames. Navigation is defined as determination of position and velocity components of a moving vehicle (Jekeli, 2001), so the navigation can be done in different frames by double integration of accelerations. Wolf (2000) and Su (2000) used the inertial frame for ephemeris computations and orbit determination of different types of satellites. Schäfer (2000) also used the numerical integration and orbit optimization using the Kalman filter on the CHAMP satellite in the inertial frame. Eshagh and Najafi Alamdari (2004 and 2005) used different methods of integration in orbit determination; they used the inertial frame in order to present the satellite orbit. Most persons working on orbit integration use the inertial frame because of its simplicity. The mathematical models of perturbing forces are also formulated in this frame. The output of the navigation solution is the position and velocity vectors coordinated in this frame too.

In this paper we consider the possibilities of integrating orbit in a local frame and we show that the satellite orbit can also be integrated in such frames. The local frames are the moving frames with the vehicle and the integration is directly preformed on geodetic coordinates of the vehicle. Although orbit integration in such frames is more complicated than the inertial frames, the presented formulas can easily be used in other applications of navigation. The NED frame of navigation has difficulties when the satellite is approaching the poles because of meridian convergence. This problem can be solved by selecting a wander frame.

In the next section of this paper the navigation equations in different frames are generally introduced. This section is divided into 4 sub-sections, and in each sub-section the navigation equation is formulated so that we have simpler differential equations of motion. Numerical studies related to the local frames are also presented in corresponding sub-sections. In Section 3 conclusions are also presented.

2 NAVIGATION EQUATIONS

Navigation or positioning using an inertial navigation system is based on integrating the sensed accelerations with respect to time, in order to obtain velocity and after that position vectors. It is clear that the navigation equation is a second order differential equation; such differential equation can be written as

$$\ddot{x} = g(x) + a\,, \tag{1}$$

where, x is the position vector, g is the Earth's gravity field corresponding to position vector x, a is sensed or non-gravity field accelerations acting on the vehicle. This a differential equation can also be written in other non-inertial frames, such a differential equation is known as free-inertial equations (Jekeli, 2001).

This section is divided into 4 sub-sections in which the navigation equations are formulated. In the next sub-section the navigation equations in an inertial frame are presented, and in sub-sections 2-2, 2-3 and 2-4 these equations are presented in an ellipsoidal (geodetic), north-east-west (NED) and wander frames, respectively as well as numerical studies on local frames.

2-1 NAVIGATION EQUATIONS IN AN INERTIAL FRAME

The Newtonian definition of the inertial system implies an Euclidean system, defined as a system with coordinates satisfying Euclidean geometry. The dynamics of the motion in this system can be formulated on the basis of Newton's second and third laws. We will retain the name inertial frame for the frame that is attached to the Earth's center of mass and is non-rotating. The frame's orientation is fixed to the celestial sphere as realized by the observed directions of quasars, extremely distant celestial objects that have not shown any evidence of changing their relative orientation. The dynamic equations of motion of a vehicle in an inertial frame can be expressed as follow

$$\frac{d}{dt}\dot{x}^i = a^i + g^i\,, \tag{2}$$

$$\frac{d}{dt}x^i = \dot{x}^i\,, \tag{3}$$

where, x^i and $\dot{x}^i$ are the position and position rates, and a^i and g^i are the measured and gravitational accelerations in the inertial frame, respectively. This is usual method of integration of the equation of motion of any vehicle like a satellite as employed by Wolf (2000), Su (2000). Most of the perturbing forces and their mathematical relations are formulated in such a frame. This is why this frame is of interest to most of the researchers. Numerical integration algorithms can be used for solving the navigation equations easily in this frame. Many numerical integration methods exist for solving such differential equations, such as Runge-Kutta (Babolian and Maleknejad, 1995) algorithms of different orders, Adams-Bashforth, and Adams-Moulton predictor-corrector algorithms (Babolian and Maleknejad, 1995), Runge-Kutta Fehlberg, or Adam step-variable numerical orbit integration methods (Eshagh, 2005). In order to solve these equations the algorithms of Runge-Kutta of the fourth order are employed in this study; see e.g. Eshagh (2003).

The problem of initial values is of importance for integrating the equations of motion. In order to start the integration process we use the initial values due to the ESA (G. Plank, personal discussion, European Space Agency) of the GOCE (Gravity field and Ocean Circulation Explorer) satellite. For more details about how to integrate the orbit in this frame the interested reader is referred to Eshagh and Najafi Alamdari (2006).

2-2 NAVIGATION EQUATIONS IN AN ELLIPSOIDAL FRAME

An ellipsoidal frame is fixed to the Earth; its origin is also at the Earth's center of mass. Its coordinate's axes are defined by convention such that the 3-axis is a mean, fixed, polar axis; and, on the corresponding equator a zero-longitude is defined that specifies the location of the 1-axis. The general dynamic equations of satellite motion in the ellipsoidal frame have been presented in Jekeli (2001) in matrix form and we have summarized those equations as

$$\frac{d}{dt}\begin{bmatrix}\dot{x}_1^e\\ \dot{x}_2^e\\ \dot{x}_3^e\\ x_1^e\\ x_2^e\\ x_3^e\end{bmatrix} = \begin{bmatrix}-2\omega_e\dot{x}_2^e + \omega_e^2 x_1^e + a_1^e + g_1^e\\ 2\omega_e\dot{x}_1^e + \omega_e^2 x_2^e + a_2^e + g_2^e\\ a_3^e + g_3^e\\ \dot{x}_1^e\\ \dot{x}_2^e\\ \dot{x}_3^e\end{bmatrix} \tag{4}$$

and ω_e is the Earth's rotation rate, $\dot{x}^e$ and x^e are the velocity and position vectors in the ellipsoidal frames. This set of differential equations can easily be solved using different integrators, like Runge-Kutta of the fourth order. It is of importance to note that the Earth's gravity field and acceleration components should be given in the ellipsoidal frame before integration. This frame is useful to integrate the orbit in existence of just geopotential perturbations.

2-3 NAVIGATION EQUATIONS IN A NED FRAME

A NED frame is a special type of the local coordinates systems; where the third axis is aligned with the ellipsoidal normal at a point, in the "down" direction, the first axis points due north, and the second axis points east. The NED frame, adopted here and conventionally implemented in the field of inertial navigation, is known as the navigation frame. Its 3-axis does not pass through the Earth's center of mass; this is the compilation of transformation between this frame and the ellipsoidal frame. The purpose of this frame is to provide local directions, north, east, and down along which velocities may be indicated. This is particularly useful in those navigation systems that are mechanized such that the sensors are always aligned with the local horizon and the vertical. The NED frame serves to define local directions for the velocity vector determined in a frame in which the vehicle has motion. The original formulas of the navigations in this frame are in Jekeli (2001) and we summarized it them as:

$$\frac{d}{dt}\begin{bmatrix} v_N \\ v_E \\ v_D \\ \phi \\ \lambda \\ h \end{bmatrix} = \begin{bmatrix} a_N - 2\omega_e \sin\phi v_E - \dot{\phi} v_D - \dot{\lambda}\sin\phi v_E + \bar{g}_N \\ a_E + 2\omega_e(\cos\phi v_D + \sin\phi v_N) + \dot{\lambda}(\sin\phi v_N + \cos\phi v_D) + \bar{g}_E \\ a_D - 2\omega_e \cos\phi v_E - \dot{\phi} v_N - \dot{\lambda}\cos\phi v_E + \bar{g}_D \\ v_N/(M+h) \\ v_E/(N+h)\cos\phi \\ -v_D \end{bmatrix}, \qquad (5)$$

where φ, λ and h are latitude, longitude and geodetic height, respectively, v_N, v_E and v_D are the components of the velocity vector in the NED frame. The a_N, a_E and a_D are the components of the acceleration vector and g_N, g_E and g_D are the components of the Earth's gravitational vector. Integration of the above equations is unstable in the vertical channel. Therefore, over longer periods it is attempted to control the error due to the vertical components, for example by altimetry measurements in airborne aspects (Jekeli, 2001). Another problem of the above system of differential equations is the initial values. The initial values can be obtained by an external source like GPS receiver carried by the vehicle or satellite.

Let the satellite is integrated using Runge-Kutta algorithm of the fourth order in the NED frame. The initial values should also be transformed into the NED frame before integration. Using the measured accelerations and these initial values and above system of differential equations we have figures 1 (a), (b), and (c) show the latitude, longitude, and height of the satellite in the NED frame. As can be seen, the satellite is unstable in the height channel when it is approaching the poles, and it is unbounded. Similar interpretation can be expressed for the velocity according to figure 1 (d), (e) and (f). The other components seem to be stable. It is better to say that the NED frame should be used for very low duration of flight or satellite revolution. It is not suitable for longer period of integration; because the unbounded behavior of vertical channel makes the solution more unstable as time increase and it can affect other parameters like longitude after long time. This frame is not suitable for orbit integration of GOCE, in spite
suitable for orbit integration of GOCE, in spite
of this fact that this frame is very useful for gradiometric measurements (Rummel et al. 1993).

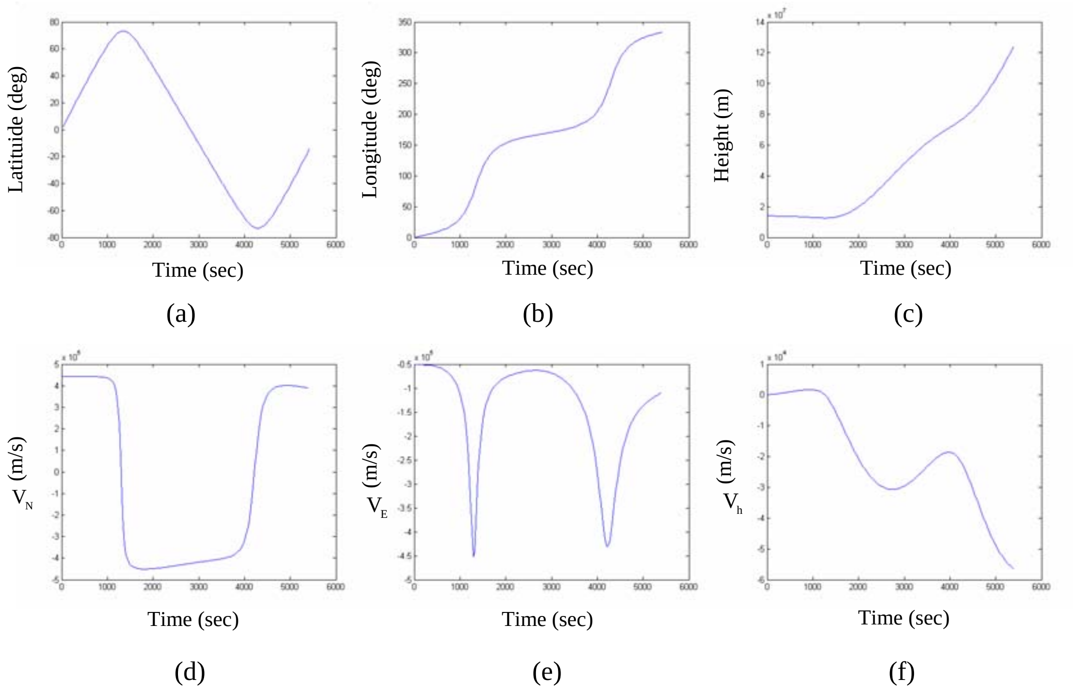

Figure 1. Behaviour of the satellite state components in the NED frame, (a) is latitude, (b) longitude, (c) geodetic height, (d) V_N, (e) V_E and (f) V_h.

2-4 NAVIGATION EQUATIONS IN A WANDER FRAME

A wander frame differs from the NED frame just with an angle called the wander azimuth. As was mentioned in the paper the integration in the NED frame become unstable at the poles, because of the convergence of meridian. The z-axis of this frame is north-ward direction of normal to the reference ellipsoid, the x-axis of this frame is rotated by the wander azimuth in order to avoid the singularities, and the y-axis is perpendicular to the both other axes. The equation of motions of a vehicle in the wander frame has the following form

$$\frac{d}{dt}v^w = a^w - \left(\Omega_{iw}^w + \Omega_{ie}^w\right)v^w + \bar{g}^w \quad (6)$$

where v^w, a^w and $\bar{g}^w$ are the vectors of velocity, acceleration and gravitation in wander frame, for expressing $\Omega_{iw}^w + \Omega_{ie}^w$ let us start with

$$\omega_{iw}^w + \omega_{ie}^w = \omega_{ie}^w + \omega_{ew}^w + \omega_{ie}^w$$
$$= \omega_{ew}^w + 2C_e^w \omega_{ie}^e \quad (7)$$

where $\omega_{ie}^e = [0 \quad 0 \quad \omega_e]^T$ and ω_e is the Earth's rotation rate, C_e^w is the transformation matrix from ellipsoidal to the wander frame, for more detail see Jekeli (2001). The ω_{ew}^n is written as

$$\omega_{ew}^n = \left[\frac{v_E}{(N+h)} \quad -\frac{v_N}{(M+h)} \quad -\frac{v_E}{(N+h)}\tan\varphi + \dot{\alpha}\right] \quad (8)$$

and $\omega_{ew}^w = C_n^w \omega_{ew}^n$ where C_n^w is the transformation from the NED and wander frames. After some simplifications and derivations we obtain equation (9):

$$\omega_{ew}^w = \frac{1}{(M+h)(N+h)}\begin{bmatrix} v_2^w(M(1-e'^2\cos^2\varphi\sin^2\alpha)+h) - e'^2 M v_1^w \sin\alpha\cos\alpha\cos^2\varphi \\ -v_1^w(M(1+e'^2\cos^2\varphi\cos^2\alpha)+h) + e'^2 M v_2^w \sin\alpha\cos\alpha\cos^2\varphi \\ 0 \end{bmatrix} \quad (9)$$

where $M = a\left(1-e^2\right)/\left(1-e^2\sin^2\varphi\right)^{\frac{3}{2}}$ and $N = a/\left(1-e^2\sin^2\varphi\right)^{\frac{1}{2}}$, a is the semi-major axis of the reference ellipsoid and h is the geodetic height, φ is latitude, e^2 and e'^2 are the first and second eccentricities and finally α is the wander azimuth. In these derivations we use $\dot{\alpha} = v_E \tan\varphi/(N+h)$ as a differential equation for the wander azimuth which is called the free-azimuth approach (Jekeli, 2001). There are two other methods for computing the wander azimuth, like Focault and Unipolar (Kayton and Fried, 1997). We have used the free-azimuth approach because of its simplicity. Now we can write $\omega_{iw}^{w} + \omega_{ie}^{w} = \omega_{ew}^{w} + 2C_e^w\omega_{ie}^{e} =$

$$= \begin{bmatrix} \dfrac{v_2^w(M(1-e'^2\cos^2\varphi\sin^2\alpha)+h) - e'^2 M v_1^w \sin\alpha\cos\alpha\cos^2\varphi}{(M+h)(N+h)} + 2\omega_e\cos\alpha\cos\varphi \\ \dfrac{-v_1^w(M(1+e'^2\cos^2\varphi\cos^2\alpha)+h) + e'^2 M v_2^w \sin\alpha\cos\alpha\cos^2\varphi}{(M+h)(N+h)} - 2\omega_e\sin\alpha\cos\varphi \\ -2\omega_e\sin\varphi \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \quad (10)$$

The final differential equations will be

$$\frac{d}{dt}\begin{bmatrix} v_1^w \\ v_2^w \\ v_3^w \end{bmatrix} = \begin{bmatrix} a_1^w \\ a_2^w \\ a_3^w \end{bmatrix} - \begin{bmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & A_1 & 0 \end{bmatrix} \times \begin{bmatrix} v_1^w \\ v_2^w \\ v_3^w \end{bmatrix} + \begin{bmatrix} \bar{g}_1^w \\ \bar{g}_2^w \\ \bar{g}_3^w \end{bmatrix} \quad (11)$$

After substituting equation (10) into equation (11) and after simplification we have

$$\frac{d}{dt}\begin{bmatrix} v_1^w \\ v_2^w \\ v_3^w \\ \varphi \\ \lambda \\ h \\ \alpha \end{bmatrix} = \begin{bmatrix} a_1^w + 2\omega_e(v_3^w\sin\alpha\cos\varphi - v_2^w\sin\varphi) + \dfrac{v_3^w v_1^w}{(N+h)} - \dfrac{Me'^2\cos^2\varphi\cos\alpha(v_2^w\sin\alpha - v_1^w\cos\alpha)}{(N+h)(M+h)} + \bar{g}_1^w \\ a_2^w + 2\omega_e(v_1^w\sin\varphi + v_3^w\cos\alpha\cos\varphi) + \dfrac{v_3^w v_2^w}{(N+h)} - \dfrac{Me'^2\cos^2\varphi\sin\alpha(v_2^w\sin\alpha + v_1^w\cos\alpha)}{(N+h)(M+h)} + \bar{g}_2^w \\ a_3^w - \dfrac{(v_1^w)^2 + (v_1^w)^2}{(N+h)} - \dfrac{Me'^2\cos^2\varphi\left((v_1^w)^2\cos^2\alpha - (v_2^w)^2\sin^2\alpha + 2v_1^w v_2^w\sin\alpha\cos\alpha\right)}{(N+h)(M+h)} + \bar{g}_3^w \\ (v_1^w\cos\alpha - v_2^w\sin\alpha)/(M+h) \\ (v_1^w\sin\alpha + v_2^w\cos\alpha)/(N+h)\cos\varphi \\ -v_3^w \\ -\dfrac{(v_1^w\sin\alpha + v_2^w\cos\alpha)}{(N+h)}\tan\varphi \end{bmatrix} \quad (12)$$

Equation (12) is the system of dynamic equations of motion of any vehicle in a wander frame. This set of differential equations can be solved numerically using any numerical integrators, such as Runge-Kutta, Runge-Kutta-Nystrom (Shidfar, 1994), Adams, Gauss-Jackson or Cowell (Santos, 1994) and so on. In order to perform simple numerical studies, the satellite is also considered. As we saw, because of the singularities of integration in the NED frame especially at the poles, we should define the wander frame. In this section we will show that the singularities of the NED frame will be removed by using this frame. Let us consider the state vector of the vehicle in one revolution.

Figures 2(a) and (b) are the satellite's latitudes and longitudes in degree, figure 2(c) is the satellite geodetic height in meters, figure 2(d), (e), and (f) are the velocity components of the velocity vector in the wander frame, and the last figure shows the behaviour of the wander azimuth in this frame. The latitude and longitude have the same behaviour as the results of integration in the NED frame, but bounded treatment for the vertical channel of h and V_3^w can easily be seen, it means that the wander frame is able to remove the singularities due to the convergence of the meridians. The interesting point is the variation of wander azimuth in order to escape the singularities; this angle varied within 160°. It may be due to the polar gaps of the satellite orbit.

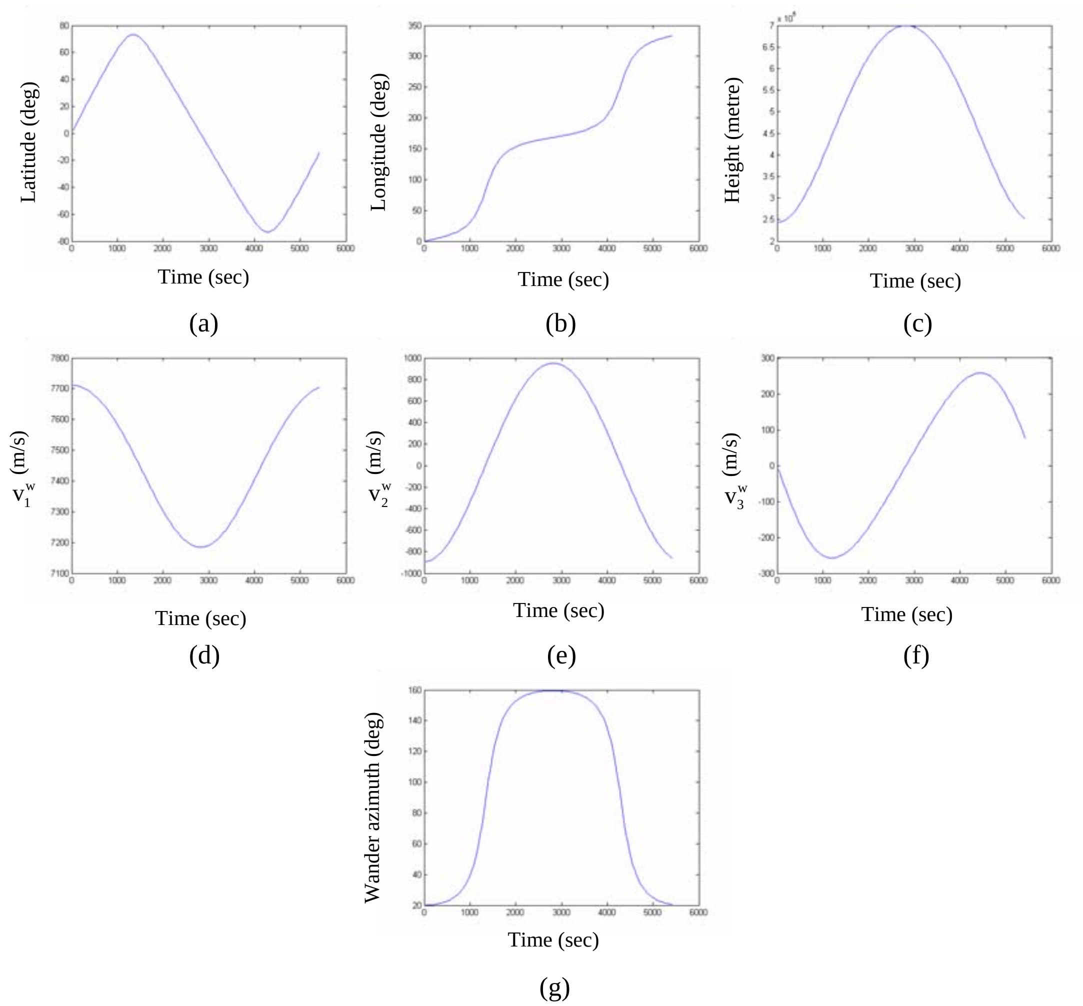

Figure 2. Behaviour of GOCE satellite components in w-frame, (a) latitude, (b) longitude, (c) height, (d) V_1^w, (e) V_2^w, (f) V_3^w and α wander azimuth.

3 CONCLUSIONS

In this paper the equation of motion of a satellite was presented in different frames of navigation. The inertial frame is a common frame being used by many persons working on orbit determination. The NED and wander frames are mostly used for aircraft navigation. Due to the meridian convergence the NED frame is not suitable for the polar region, because in this case singularity will happen in the solution. However, this frame can be employed for aircraft navigation for short duration flights and non-polar regions. The vertical channel of the integration usually behaves unboundedly. In aircraft navigation external sources are suggested for use in order to bound this treatment, usually an altimeter is useful for this purpose. Since a satellite passes twice over the poles the solution of the navigation equations becomes singular in these regions. This singularity affects the vertical bounds of navigation too, so that after some revolutions the horizontal bound also deteriorates. Our computations show that such problems can be removed in the integration process using a wander frame; the wander frame introduces a wander azimuth in order to overcome singularities due to the meridian convergence, our computations show variations of about 160 degrees in the wander azimuth during orbit integration. No singularity happens in this frame even in the polar region and flights of long duration. It is suggested that the wander frame be used whenever one wants to integrate the satellite orbit in a local frame. Although we presented the satellite's motion in such a frame, it is always possible to integrate the satellite orbit in an inertial frame and transform it in another frame. However, our main goal was to present the wander frame ability.

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